MULTILINGUAL MATHEMATICS TEACHING AND LEARNING: LANGUAGE DIFFERENCES AND DIFFERENT LANGUAGES

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The reported research provides findings from the study of lesson episodes during student group work in multilingual mathematics teaching and learning. The theoretical lens of language-as-resource is taken to examine how certain uses of language and representations of speakers are voiced in ways that positively mediate the emergence and restoration of mathematics learning opportunities. This is illustrated with an example of analysis applied to an episode where language concerns are built up close to the communication of a mathematical idea for the resolution of a task. Overall, language is framed as a powerful resource in the classroom, whose resourcing for mathematics learning implies a multiplicity of languages (and hence discourses and voices) about language modeling and group identification.

WHAT KIND OF RESOURCE IS LANGUAGE?

In this report, the notions of social language and mathematics learning opportunities are examined under the theoretical lens of language-as-resource (Planas, 2014). This lens presupposes the ontological stance that language is not an actual resource unless someone uses it in a context of activity with particular tasks intended for some learning opportunity to emerge. It is therefore in the use that the potential quality of resource for a certain purpose can be realized and recognized (Remillard, 2013). Once language is put to use in context, it is also presupposed that numerous possible directions for the development of activity are present in a number of ways, each of them of value within specific discourses and for the resourcing of particular directions. In this framework, the issue of how teachers and students use language as they do during multilingual mathematics teaching and learning becomes fundamental.

To examine this issue, in my classroom-based research I look at how social languages and mathematics learning opportunities work together in the understanding of small group work and whole class discussion. In the research completed so far with group work episodes in four lessons, the analysis shows discourses about uses of language and representations of speakers in interaction with the course of student mathematical activity. At this stage a case can be argued regarding the relationship between the creation and restoration of mathematics learning opportunities, on the one hand, and what is discursively built up and voiced in classroom discourse with the support of a variety of languages and their speakers, on the other.
Putting language into the social

Previous work on students’ difficulties with languages has been decisive in setting a rationale for pursuing a profound understanding of language (diversity) in mathematics education research (Phakeng, 2016). Forms of speaking, varieties of languages, discourses and voices constitute a family of notions that have preceded and prepared adoption in the field of the notion of social language and, with it, the progressive move away from deficit-based arguments. In this respect, Barwell (2016) draws attention to how “natural” languages (i.e. normative vocabularies tied to abstract grammatical systems) weave discourses and voices together, and to how discourses and voices in turn weave representations of certain languages as “natural.” Mathematics teaching and learning is rarely about language in the broad sense in which we talk about using, e.g., Spanish, but rather about the “social languages” that recognizable groups of people use to carry out and voice their social practices. When we enter the mathematics classroom, we all navigate within, between and across the different social languages through which participants express their views and worlds.

The notion of social language calls into question developmental views of language proficiency as a variable of time and individual effort, as well as essentialist views of language diversity that equate, e.g., one bilingual classroom with two distinct languages. The illusion of measuring language proficiency and labeling and counting languages indicates the underlying conceptualization of language as material, pure and unitary. Even though this may not be the standpoint taken in some research, exclusive expressions like bilingual classrooms and bilingual students are not rare. Languages may look more alike if they exist within a “single” labeled language, and indeed this is relevant in the understanding of the actual diversity of a classroom, but there is a more complex reality across the countless social languages – that often go under the rubric of a language – with a role in representing student (language/activity) proficiency.

Situating language in mathematics learning

The question of the social underpinnings of mathematics learning is not new. In their seminal work, Yackel, Cobb and Wood (1991) presupposed the availability of opportunities for the learner to learn mathematics; that is the existence of mathematical ideas and social conditions more or less ready to be grasped for the development of mathematics learning in a context of activity. The research design experiments that followed from that work aimed to introduce changes in the social conditions of teaching and learning in mathematics classrooms. These experiments were substantiated by three inseparable claims: learning cannot take place without learning opportunities being available, these opportunities are created by people, and they are made available in accordance with the social conditions and not only the personal insights of individuals. Since not all the opportunities created in a context of activity are tackled as such by everyone all of the time, a separate issue was whether or not they are exploited in activity conducive to individual learning.
Together with the readiness of favorable social conditions, Saxe (2012) relates the discussion of mathematical ideas to the creation, exploration and development of mathematics learning opportunities. He refers to the travel of ideas in his research into the ways in which mathematical ideas are produced and transformed over the course of discussion-rich interaction, hence enabling new ideas to emerge. The availability of opportunities to learn mathematics is thus posed in relation to the availability of resources to allow ideas to surface and travel. Making mathematical ideas travel implies in turn the availability of resources for participation in discussion-rich interaction, as well as the availability of opportunities to utilize these resources in classroom activity. This formulation supports the relationship between opportunities to learn mathematics and opportunities to resource mathematics learning. Given that language is critical for participation in discussions, we finally come to the connection between availability of opportunities to learn mathematics and availability of (social) language(s) in the communication and discussion of mathematical ideas.

From a developmental approach, language availability is a long-term product that, once achieved by someone, implies durability. Accordingly, some learning opportunities are thought of as diminished or postponed in contexts of activity where there are participants who do not “own” such a product and, for this reason, are expected to contribute less than others. Within the framework of social languages, language availability – and the corresponding facilities to allow ideas to be expressed, collected and commented upon by multiple people – implies a different understanding (Planas, 2014). This availability is variously high or low for a number of reasons other than preconceived levels of proficiency in a given language. Thus, it is not a product to be achieved by individuals, but a dynamic feature of the context in which language is put to use by various participants to voice different discourses in the interaction. Under the basic assumption that language is not available all the time for all participants (Makoni & Pennycook, 2005), the opportunities to use it to make ideas travel can be (dis)encouraged by infused processes of assessment and (dis)placement of speakers who do not conform to standardized forms of speaking and acting.

**APPLYING LANGUAGE-AS-RESOURCE TO THE STUDY AND VIEW OF STUDENT MATHEMATICAL ACTIVITY**

The data in this report draw from a Grade 8 classroom of a school in a low-income zone of Barcelona, the capital city of Catalonia, a north-eastern region of Spain with its own language in education policy – Catalan is the official language of teaching and learning, although it is not necessarily the language of learning and thinking for all students. The teacher was a Catalan-dominant speaker who occasionally used a variety of Castilian Spanish in her lessons. Fourteen students were children from Latin American (Colombian, Ecuadorian and Peruvian) families who declared Spanish to be their home language, nine of whom were raised abroad; five students were children of Castilian Spanish-dominant families, two of whom were raised in Castilian-speaking parts of Spain; and four were Catalan-dominant speakers raised in Barcelona. Varieties of Colombian, Ecuadorian, Peruvian and Castilian Spanish, or combinations of these,
are not typical of the varieties of Spanish spoken by people raised in Catalan-speaking regions. There are differences in the sounds of some letters and in the conjugation of some verbs, among others. Students who begin to learn the language of instruction at school – mostly due to histories of immigration – are located in special lessons for some time to learn this language. When they finally enter the regular classroom they tend to speak varieties of Catalan with sounds, conjugations and words borrowed from their home languages; such varieties are marked as “poor Catalan” by groups who claim ownership of the language of instruction in the region.

Local educational debates that occupy much of the current public discourse generally address the merits of the parallel system of special lessons for “latecomers” (Planas & Civil, 2008), rather than debating about how children learn and teachers teach in either system. Other important debates are motivated by ideological stances regarding the politics of language use at school. All these debates inform curricular decisions and pedagogic practices that often adhere to rigid conceptualizations about what counts as language – in theory and in classroom practice – and how language use is seen in relation to mathematics teaching and learning. It is thus significant to explore how language is shaped by discourses and voices that make some mathematical ideas more likely to travel (and thus some mathematics learning opportunities more likely to emerge) when they are expressed in the standardized language of instruction by speakers who are represented as (more) competent in this language. Even if a student has the necessary school mathematics knowledge to discover and value a learning opportunity, she may fail to do so because there is limited access to certain forms of speaking and speakers in the context in which the opportunity arises.

**Lesson, task and methods**

Student work in three small groups was video-taped during a problem-solving unit of four lessons devoted to algebra. The groups had one or two Catalan-dominant speakers each and remained the same throughout the sequence. For this report, I take lesson four and the group with Maria and Ton, from Catalan-dominant families, and Ada and Leo, who were raised in Peru and attended classes for latecomers during Grade 6. The problem was a representation of the Fibonacci numbers starting at 1 and 2:

In a house there is a staircase with ten steps. If we can go down the steps one or two at a time, in how many different ways can we go down the staircase?

Transcripts of lesson data were produced regardless of shifts between the two labeled languages involved. Group work was coded under three main types: Language Modeling (LM), Group Identification (GI) and Mathematical Ideas (MI). LM was assigned to turns with visible references to vocabulary (LM-V), grammar (LM-G) or pronunciation (LM-P), whereas GI was assigned to turns with mentions of or allusions to speakers as members of groups, in some of which issues of vocabulary (GI-V), grammar (GI-G) or pronunciation (GI-P) were mentioned. Each coded turn was interpreted as a component part of the activity that was constituted during mathematics teaching and learning. The segmentation of talk into turns was followed by
segmentation into instances. In this study an instance is defined as a range of spoken turns, which are sequential but may not be consecutive, from a single sentence all the way to a lengthy interaction. After the detection of LM and GI turns and the construction of paired instances, the analysis went on with the focus on student mathematical activity. This stage was guided by the search for turns and paired instances with literal comments and implied allusions regarding ideas of relevance for the understanding and resolution of the problem. MI codes were named after topic identification in accordance with the central mathematical content to be kept for the development of the idea. From here, LM, GI and MI instances were related in the construction of episodes with one MI and at least one LM/GI instance as well as sufficient before-and-after-turns to understand what was being mathematically developed and how some language issues had been voiced in-between.

For each episode and when feasible, relationships were elaborated between the availability of language and the availability of mathematical ideas. This was done by imagining “figured worlds” (Holland, Lachicotte, Skinner & Cain 1998) under two basic phenomenological standpoints: 1) any account of reality requires imagination, and 2) imagination is necessary to make any inference out of what appears. In the application of imaginative variation to each episode – any variation is a possibility –, the following dual question was posed: ‘Can language modeling and group identification be imagined as obstacles to/resources for mathematics learning?’ Answers originated from the examination of possible directions in the ways that discourses and voices (could) put language to use. This process served to imagine language within a cycle of diverse resourcing directions. Since many interpretations can be imagined, the variations considered were only those in which the new episodes lacked the coded language turns but kept the mathematical idea. The varying of language modeling and group identification was undertaken to explore how realistic the development of the corresponding mathematical idea was in the worlds imagined.

“Minor” details taking on “major” importance

In this section, we analyse an episode of group work in lesson four. The creation of certain opportunities to learn from the study of a simpler version of the given problem is associated with the voicing of some concerns about language modeling (LM-V) and group identification (GI-V). Language is interactionally worked out turn by turn in ways that model forms of speaking, represent groups of speakers, and allow the emergence of new ideas to be discussed. The transcript below shows how apparently “minor” details in language use can take on “major” importance in interaction. Language modeling and group identification are voiced, respectively in relation to the right meaning of the Catalan word for going down (“baixar” in [5]) and the name of the student at risk of interpreting the word wrongly (“Ada” in [6]). When it is later said that there is “too much to go down and jump” [9], mentions of the distinction between the two verbal actions are not taken up and a relevant mathematical idea for an approach to the resolution of the Fibonacci problem emerges instead.
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1 Maria: Per què tens tot uns i aquí tot dosos? [Why do you have all ones and here all twos?]

2 Leo: Puedes bajar siempre o saltar siempre. [You can always go down or always jump.]

3 Maria: Sempre es baixa, no t’estàs parat. [You always go down, you don’t stand still.]

4 Leo: Pero a veces no bajas, saltas. Y a veces solo bajas. [But sometimes you don’t go down, you jump. And sometimes you go down only.]

5 Ton: Baixar no vol dir d’un en un. Mira, baixar és un a un, dos a dos, tres a tres, tot és baixar. [Going down does not mean one by one. Look, going down is one at a time, two at a time, three at a time, all this is going down.]

6 Maria: Ada, tu ho tens clar? [Ada, is this clear to you?]

7 Ada: Sí, baixar. [Yes, going down.]

8 Ton: Així et deixes de barrejar uns i dosos. [This way you miss combinations of ones and twos.]

9 Leo: He empezado pero hay mucho que bajar y saltar. Al menos treinta. Si la escala fuera más corta… [I began but there is too much to go down and jump. At least thirty. If the staircase was shorter…]

10 Ton: Umm… Si fos tres, seria: u, u, u; dos, u; u, dos… i dos, dos impossible. Ara ve quatre. [Umm… If it was three, it would be: one, one, one; two, one; one, two… and two, two impossible. Now four comes.]

There is some difficult mathematics involved in the resolution of the problem in this lesson. One can always determine the possibilities by counting them one by one, but this is not very manageable, as suggested by Leo in [9]. While it is easy to represent the extreme cases [1], when the combinations of ones and twos are considered in a classroom with students who are not familiar with combinatorial formulas and binomial coefficients, a process to represent the total of 89 possibilities is not easy to discern; it may occur that one possibility is counted twice or that some possibilities are missed during the counting. Nonetheless, there is a pattern embedded in the resolution whose exploration can be strategically approached by starting with staircases which have smaller numbers of steps (the 3-step and the 4-step staircases in [9-10]). Although the students from the group did not see a pattern, they foresaw the option of examining reductions of the problem and, hence, approached the challenge of solving the problem without adding up the total number of ways of going down ten steps. At the end of the lesson, the teacher presented the recursive pattern that relates the number of ways to get the 10th step to the number of ways to reach the 8th and the 9th, and successively until the dependence of the 10-step on the 1-step and the 2-step staircases.

The mathematical idea introduced by Leo in [9] and taken up by Ton in [10] is preceded by two moments in which activity moves away from the task resolution toward language concerns. Maria and Ton model the acceptable meanings for a term in Catalan when Leo equates the movements of one step at a time with "baixar" (going
down) and two steps at a time with “saltar” (jumping). In [3] and [5] “baixar” is given an extended meaning that includes jumping, which is the common meaning in mainstream Catalan. Another mathematically critical moment comes when group identification is voiced in [6]. The suggestion that Ada may experience the same confusion with vocabulary, as she may be interpreting “baixar” like Leo, can be seen as an allusion to the qualities attributed to the group of people that Leo and Ada are seemingly placed in. References to shared backgrounds are echoed and the account of these students as poor users of the official language of teaching and learning is made visible. However, when the distinction between going down and jumping comes again in [9] during the proposal of the idea about shorter staircases, discourse moves away from the focus on language issues and goes back to mathematics.

What we see in this episode is that language modeling and group identification are voiced in ways that momentarily interrupt the mathematical discussion in student group work. However, it is from here that another discourse emerges with the option for the students to unvoice vocabulary and group differences and explore a more sophisticated approach to the resolution of the problem. A primary representation of language as obstacle is thus difficult to imagine in this episode, as is a primary representation as epistemological resource for mathematics learning. By varying [5] and [6] and imagining an alternative episode without these turns, the possibility of the same mathematical idea emerging and traveling with the same intensity in student interaction is feasible. This said, important learning opportunities arise from the fact that the participation of Leo is facilitated and recognized in a discourse that follows the anticipation of language difficulties and differences among students.

**DIRECTIONS IN THE RESOURCING OF LANGUAGE**

In this report, by means of a short transcript, I have tried to illustrate the diversity of directions in the resourcing of language. The finding regarding the plurality of directions in the resourcing of language during student mathematical activity provides further understanding about the role of language in mathematics teaching and learning, especially as it intersects with a multiplicity of languages in the multilingual mathematics classroom. The discussion of social languages suggests that, far from assuming that language and mathematical difficulties reside in students, it is reasonable to consider that some of these difficulties as well as the possibilities of overcoming them primarily reside in discourse. In the data presented, Leo and Ada are immigrant children from lower income homes who have been taught a differentiated mathematics curriculum during their school year in the parallel system of special lessons for “latecomers.” They have little practice at home with school-based forms of language and interaction, and they are actually represented as poor speakers of the official language of instruction. Nonetheless, we have seen how Leo participates with a relevant mathematical idea in the middle of a discourse in which language difficulties and differences are voiced. Both the relevance of the distinction between ‘going down’ and ‘jumping’ and the relevance of Leo’s mathematical idea reside in discourse.
The learning of mathematics cannot be understood separately from the process of learning the social language that is characteristic of those who do well in the school mathematics of the local educational system. All children can effectively be introduced to the dominant social language of a given classroom. Nonetheless, this should not be done at the expense of reducing their participation in the creation of mathematics learning opportunities during the process of learning a language. In the teaching and learning it is not easy to juxtapose the primary languages with the newer languages so as to allow students to smoothly navigate across them for the primary purpose of mathematics learning. Hence, the importance of intentionally integrating in mathematics teaching the issue of focusing on the many mathematical ideas that can be communicated and discussed at the intersection of the social languages that students and teachers bring with them. It cannot be forgotten that the resourcing of language for multilingual mathematics teaching and learning rests not only upon the possibility of resourcing the emergence, exploration and development of mathematical ideas, but also upon the possibility of postponing the discussion of these ideas.

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