Language Diversity Builds Mathematics Learning as Much as Mathematics Learning Builds Language Diversity

Núria Planas

Abstract The main idea behind the title of this chapter is that there is something about language diversity which is tied to mathematics learning in a way that justifies a detailed investigation. I take the theoretical lens of language-as-resource to examine how certain uses of language and representations of speakers are voiced in ways that positively mediate the emergence and restoration of mathematics learning opportunities. Overall, I frame language as a powerful resource in the mathematics classroom whose resourcing for mathematics learning implies a multiplicity of languages (and, hence, discourses and voices) about language modeling and group identification. This position statement goes beyond the frames of mathematics education research on language and provides new ways to think about language, what we do with it in research and practice, and why.

Keywords Language - Mathematics classroom - Mathematics learning - Multilingualism

1 Introduction

This chapter has two objectives. The first is to present an overview of literature and other findings related to multilingual mathematics teaching and learning situations. The second objective is to argue about the multilingual quality of any site of mathematics teaching and learning. It is broadly accepted that language is important for learning and thinking. Further, the ability to communicate mathematically is central to learning and teaching school mathematics (Planas et al. 2018). The new insight in mathematics education research is the recognition of multilingualism as a

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common feature of all mathematics classrooms. Part of what we have to acknowledge is that mathematics teaching and learning take place in social landscapes of multiple languages (Barwell et al. 2016). The underlying focus of this knowledge is set on what is communicated and how in the mathematics classroom. The same focus underlines the discussion throughout the chapter.

In Catalonia, my major research setting, social languages are mostly realized at the intersection of two labeled languages, Catalan and Spanish. The term labeled refers to how a language is considered at the level of policies and laws in a country. Moreover, it includes the mainstream ideologies and public discourses which traverse communities and groups. Thus, multilingualism can be interpreted as referring to the countless social languages in any classroom, even though they may be produced within a few labeled languages only. Consequently, some of them may look more alike on the linguistic level. The languages of mathematics used by students and teachers in a classroom add even more complexity to the understanding of multilingualism. This is not to suggest that some languages are mathematical and others are not. Rather, some are produced as mathematical in discourse, possibly because they look more akin to how mathematicians are expected to speak.

To begin, I introduce two basic notions from the literature related to multilingual mathematics teaching and learning situations: social languages and mathematics learning opportunities. In order to argue that multilingualism is a quality of any mathematics classroom, I examine the issue of how teachers and students use language (and, hence, discourses) as they do during mathematics teaching and learning situations. I look at how the revised notions work together in the understanding of the languages involved in two episodes of a lesson. The example of the chapter is taken from a mathematics classroom in an urban secondary school of Barcelona. Through critical discourse analysis, the multilingual use of language is linked to the production and communication of mathematical ideas. Finally, I construct a case to analyze the relationship between the construction of mathematics learning opportunities and what is discursively built up in classroom discourse with the support of a variety of languages and their speakers.

2 Which Notion of Language and for What?

Central to socio-cultural-political theories is viewing the mathematics classroom as a social space converted into a place in which groups of people come together to engage with processes of transformation (Jablonka et al. 2013; Planas and Valero 2016). These processes include mathematics learning and, more generally, changes with respect to forms of speaking and belonging. Thus, a key issue is to understand the possibilities of language for the production of forms of speaking and belonging that are recognized and valued in the mathematics classroom. This understanding includes the critical observation of situations where students from groups which benefit from certain forms of speaking and discourses are placed alongside, and interact, with students from groups which are disadvantaged by these same forms and discourses

(Wagner and Herbel-Eisenmann 2008). Some students may have to make a greater effort to act out certain languages, not only due to prior distribution of knowledge, but also due to what is communicated about them and their languages.

In Catalonia, discourses of access to the language of instruction are dominated by discourses of remedying. Such discourses of remedying, however, can be seen to undermine the possibilities of producing the kind of access required for the groups that are meant to be “helped”. Educational debates that occupy much of the current public discourse address the merits of the parallel system of special lessons for “latecomers” – who are often conflated into a generalized group of immigrant people –, with less attention paid to how students learn and teachers teach in either the regular, or the special school system. Other ongoing debates are motivated by ideological stances regarding the politics of language in formal education in regions where Catalan is the official language of teaching and learning, although it is not necessarily the language of learning and thinking for all students. All these debates and policies inform curricular decisions and pedagogies that often adhere to rigid conceptualizations of language in mathematics teaching and learning.

2.1 Putting Language Into Discourse

The notion of social languages is a theorization that embraces the more general notions of language and discourse. I assume the ontological stance that language necessarily implies language use in discourse. Like Gee (2004), I refer to discourses as representations of socially, culturally, and historically influenced ways of action and interaction which constitute an essential component of any context of language use. Such representations spread within and across languages and move through language groups.

Previous work on students’ difficulties with languages has been decisive in setting a rationale for pursuing the theoretical articulation of language and discourse in mathematics education research. In this respect, forms of speaking, varieties of languages, discourses, and voices constitute a family of notions that have preceded and prepared adoption in the field of the notion of social languages. The reworking of Gee’s theory of identity (2004) by Setati (2005, 2008), who at an early stage elaborated on the social quality of language in multilingual mathematics classrooms, is a precedent of the attention to social languages and the progressive move away from deficit-based arguments. Gee (2004) claims that languages are ideological creations of groups that exist in the form of several ways of speaking about a particular topic in a particular way at a particular place in time. It is through the use of languages that people enact socially-situated identities and identify appropriately-related activities. Thus, students are multi-discursal people in the process of enacting forms of speaking in school mathematics on their way to “becoming educated” with language capitals that can take them closer to what is (re)produced as appropriate in the classroom.
More recently, Barwell (2014, 2016) drew attention to how language systems (i.e. normative vocabularies tied to abstract grammatical systems) weave discourses and voices together. Further, the attention was shifted to how discourses and voices in turn weave representations of certain languages as “natural.” This integration of certain major contemporary research traditions concerning language diversity in the field has formed a basis for establishing a more nuanced notion of social language. Barwell takes a Bakhtinian orientation to reinforce language as dialogical and multiple (Bakhtin 1981) in landscapes of “superdiversity” (Vertovec 2007, in Barwell 2016, p. 25). When we engage in dialogue, we navigate within, between, and across discourses to face the challenges embedded in the ability to view the worlds of others. Each world reflects superdiversity in a unique way and produces vocabularies and grammars permeated by discourses through which people speak and take on certain identities. Thus, one cannot possibly expect someone to communicate the fiction of “natural” languages without a series of back-and-forth movements in discourse.

Social languages own linguistic and discursive features in their spoken and written forms that comment on multiple levels of meaning about the significance of what is communicated and how. These features allude to the pervasive presence of discourses operating at the macro level through a number of cultural and political tools such as curricula, language policies, and teaching pedagogies. Nonetheless, the production of significance cannot be understood separated from the actual discourses and features of social languages realized in the classroom (Planas and Setati-Phakeng 2014). Language capitals and language systems are reproduced, but also transformed, in the course of multilingual mathematics teaching and learning situations. All these standpoints call into question developmental and essentialist views of language. The created illusion – in theory and in classroom practice – of measuring language proficiency and counting languages indicates an underlying ideology of language as a material system of ideal speakers (Makoni and Pennycook 2005).

### 2.2 Putting Language Into Mathematics Learning

The question of the social underpinnings of mathematics learning is not new. Yackel et al. (1991) related this learning to the social conditions in a given context of mathematics teaching and learning. They coined the term of mathematics learning opportunities to express the consideration of the social. The design experiments that followed from that work aimed to introduce changes in the social conditions of teaching and learning in mathematics classrooms (Cobb and McClain 2004). These experiments were substantiated by three claims. Firstly, learning cannot take place without learning opportunities being available, secondly, these opportunities are created by people, and thirdly, they are made available in accordance with the social conditions – thereby, they are not only the personal insights of individuals. Since not all of the opportunities created in a context are treated as such by everyone all of the time, an issue that arises is whether or not they are exploited in activity conducive to individual learning. On the other hand, since not all contexts of language use develop discourses of student collaboration, not all the conditions of group work and conversation are likely to produce opportunities with the same intensity. Therefore, the study of opportunities is a way to link the social aspects of classroom activity to the students’ conceptual development around mathematical ideas without ignoring the conditions in which communication and collaboration are produced.

Saxe (2012) analytically relates the discussion of mathematical ideas to the emergence, exploration, and development of mathematics learning opportunities. This author uses the metaphor of the travel of ideas to refer to research into the ways in which mathematical ideas are (re)produced and transformed over the course of discussion-rich lessons, hence, enabling new ideas to emerge. The availability of opportunities to learn mathematics is thus posited in relation to the availability of resources to allow ideas to surface and travel. In particular, to make mathematical ideas travel implies access to and use of resources for interaction and participation. Given that language is critical for interaction and participation, we necessarily need to address the connection between mathematics learning opportunities and language. Such connection can be interpreted in light of the linguistic and discursive features of language that allude to the construction of some meanings as mathematically significant in the considered context. These meanings are produced at the level of discourse, with the developing discourses in the classroom playing a decisive role.

The proposed understanding of mathematics learning opportunities as discursive constructions is a result of the understanding of language as language use in discourse. If one adopts a developmental approach instead, language appears as a long-term product that, once achieved by someone, implies durability. Accordingly, some learning opportunities are thought of as diminished or postponed in contexts with people who do not “own” such a product. Far from this approach, the discursive perspective relates the production of learning opportunities with the facilities and difficulties for the construction of certain levels of meaning over others in language use. Neither language nor learning opportunities are products to be achieved by individuals, but developing productions of any context of language use. Under this assumption, the opportunities to use language to make mathematical ideas travel can be (dis)encouraged by infused processes of assessment and (dis)placement of some groups of speakers and some languages. What students can do, and what they actually do with language, is situated in discourses about what languages need to promote and accept to be languages of mathematics in the classroom, and about which of their languages may be more specialized to this end. The fact that a number of language capitals and language systems are always laid down in language relates the significance given to some languages of mathematics with the significance given to those who use them and their communities of reference (Jorgensen et al. 2014).
2.3 Putting Language Into Mathematics Learning in the Multilingual Classroom

At this point, it can be said that language is conveyed as a resource for mathematical learning when it is used in discourse to primarily produce and communicate mathematical ideas. Among others, this potential of language needs to be placed at the interplay of competing discourses of language systems and language capitals. In the mathematics classroom, a number of mathematics learning opportunities are thus produced in interaction with work oriented to other purposes such as the communication of the official language policy and the construction of some students in terms of learners of certain languages. Assuming that resources require work oriented to specific purposes (Pepin et al. 2013; Remillard 2013), any language use for the identification, communication, and discussion of mathematical ideas requires work centered on some of the levels of meaning in discourse.

Like language, mathematics learning opportunities are produced in the middle of discourses that make some mathematical meanings more likely to emerge when they are expressed in the normative grammars and vocabularies of the language of instruction by people with the privileged language capitals. Even if a student can produce a learning opportunity, she may fail to do so because there is limited access to certain forms of speaking in the context in which the opportunity might arise. For example, such student might be ready to communicate important mathematical meanings in a lesson about isometries, but this might be hindered by the primary communication of meanings regarding the language policy at place and the higher value given to some languages over others for the teaching and learning of isometries. This seemed to be the case in the linguistically diverse lesson of Planas and Setati (2009). Different students used formal and everyday languages of mathematics differently in combination with the language of instruction in their references to spatial transformations, compositions, arrows, and torados. In that article, some tensions are discussed in relation to pieces of language in which some students communicate the significance of contributions made through normative Catalan grammars and technical words, while some other students provide significance to everyday words, though still expressed with linguistic features of normative Catalan.

3 Context and Methods for the Study of Mathematics Teaching and Learning

The data in this chapter stem from a study carried out in a Grade-8-classroom of a school in a low-income zone of Barcelona, the capital city of Catalonia. The teacher’s primary language was Catalan, but she occasionally used varieties of Castilian Spanish in her lessons. Fourteen students in the classroom were children from Latin American (e.g., Colombian, Ecuadorian, and Peruvian) families who said that Spanish is their first language (nine of them were raised abroad), five students were children of Castilian Spanish-dominant families (two of them were raised in Castilian-speaking regions of Spain), and four were students with Catalan as their first language, raised in Barcelona. Varieties of Colombian, Ecuadorian, Peruvian, and Castilian Spanish- or combinations of these— are not typical for the varieties of Spanish spoken by people raised in Catalan-dominant regions of Spain. There are, for example, differences in the sounds of some letters (e.g., grave and acute vowels) and in the conjugation of some verbs (e.g., continuous and simple tenses). On the other hand, students who begin to learn the language of instruction at school tend to speak varieties of Catalan with sounds, conjugations, and words from their home languages. These varieties are marked as “poor” by those groups who claim ownership of the language of instruction in the region (Pujolar 2010).

The history of official language policy in Catalanian formal education is associated with the politics of repression, resistance, and affirmation of the Catalan language from 1939 to 1975 in Spain. At that time, Catalan was relegated to a lower status, and it was denied the right to be spoken outside the home. Subsequent to the first democratic elections in Spain, the Catalan Parliament voted on various laws and policies, still intact, to protect the use of Catalan. The practical implications for teaching and learning are various. In some of the socio-economically poor areas of the city, considerably high rates of immigration from Latin America have led to a majority of Spanish speaking students in schools. Given the linguistic closeness between Catalan and Spanish, and the common bilingualism in the region, Latin American “latecomers” are expected to quickly use the newer language and, in the meantime, they are designated as “learners of Catalan”. In about a year, their performance of written and oral Catalan in standardized tests is rather high. Consequently, they no longer need to attend “special lessons for latecomers” during school hours. All these terms — “latecomers”, “learners of Catalan”, “special lessons”— reflect some of the ways in which the official language policy enters schools to communicate a distinction between two groups of students in the region.

In order to collect discussion-rich classroom data, student work in three small groups was video-taped during a problem-solving sequence of four lessons devoted to algebra (see more about the investigation in Planas 2017). The groups had one or two Catalan-dominant speakers each. These remained the same throughout the sequence. In this chapter, I analyse the group consisting of Maria, Ton, Ada, and Leo during the fourth lesson. The selection of this group and lesson is deliberate because the communication of different levels of meaning in discourse is quite explicit in various pieces of language. Ada and Leo were designated like “latecomers” and “learners of Catalan” at their arrival to the school in sixth grade and remained such until the beginning of seventh grade. They were raised in Peru and said to speak Spanish at home. The other twelve-year-old students of the group, Maria and Ton, were born in Barcelona, had always attended the system of regular lessons and said to speak Catalan at home. During the school year in the special system of lessons, Ada and Leo had been with five more students of immigrant families and were taught an adapted curriculum with the main goal, literally, to facilitate the learning of the language of instruction in order to join the regular system as soon as possible. Moreover, during the time of data collection, the teacher
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one key aspect of any critical discourse analysis is the view that any situated use of language is produced under the influence of a number of discourses alive in the wider social, cultural, and political contexts. Thus, the analysis of the following examples provides insights into language policies and pedagogies of remedial mathematics education for immigrant students and how these enter the mathematics classroom.

Example 1: “If the staircase was shorter…” This first example is taken from the timespan of the lesson devoted to group work. Language modeling and group identification are identified in relation to the right meaning of the Catalan word for going down (“baixar” in [5]) and the name of the student at risk of interpreting the word wrongly (“Ada” in [6]). Furthermore, a mathematical idea regarding a solving strategy is identified throughout the explicit communication of the attention to linguistic features of the students’ languages and possibly throughout the implicit communication of linguistic normativity and language capitals.

<table>
<thead>
<tr>
<th></th>
<th>Maria</th>
<th>Per què tens tots i aquí tot dosos? [Why do you have all ones and here all twos?]</th>
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<tbody>
<tr>
<td>2</td>
<td>Leo</td>
<td>Puedes bajar siempre o saltar siempre [You can always go down or always jump.]</td>
</tr>
<tr>
<td>3</td>
<td>Maria</td>
<td>Sempre es baixa, no t’estats parat. [You always go down, you don’t stand still.]</td>
</tr>
<tr>
<td>4</td>
<td>Leo</td>
<td>Pero a veces no bajas, saltas. Y a veces solo bajas. [But sometimes you don’t go down, you jump. And sometimes you go down only.]</td>
</tr>
<tr>
<td>5</td>
<td>Ton</td>
<td>Baixar no vol dir d’un en un. Mira, baixar és un a un, dos a dos, tres a tres, tot és baixar. [Going down does not mean one by one. Look, going down is one at a time, two at a time, three at a time, all this is going down.]</td>
</tr>
<tr>
<td>6</td>
<td>Maria</td>
<td>Ada, tu ho tens clar? [Ada, is this clear to you?]</td>
</tr>
<tr>
<td>7</td>
<td>Ada</td>
<td>Sí, baixar. [Yes, going down.]</td>
</tr>
<tr>
<td>8</td>
<td>Ton</td>
<td>Et deixes de barrejar uns i dosos. [You miss combinations of ones and twos.]</td>
</tr>
<tr>
<td>9</td>
<td>Leo</td>
<td>He empezado pero hay mucho que bajar y saltar. Al menos treinta. Si la escala fuera más corta… [I began but there is too much to go down and jump. At least thirty. If the staircase was shorter…]</td>
</tr>
<tr>
<td>10</td>
<td>Ton</td>
<td>Umm… Si fos tres, seria: u, u, u; dos: u, u, dos… i dos, dos impossible. Ara ve quatre. [Umm… If it was three, it would be: one, one, one; two, one, one, two… and two, two impossible. Now four comes.]</td>
</tr>
</tbody>
</table>

4 Construction of Mathematics Learning Opportunities

In line with the second objective of the chapter, this section aims to illustrate how different languages transverse student work as well as how their study reveals the construction of multilingual mathematics learning. The production of exemplifying opportunities to learn from the reduction of the problem in lesson four (Example 1), and from the relationship of recursivity between the problem and some reduced versions (Example 2), are investigated on in relation to the production of some concerns about vocabularies, grammars, and language capitals. It is argued that language is used by students for the construction of such concerns, but also for the production of mathematical ideas of relevance within the context of the task. On the other hand,
the possibilities by counting them one by one, but this is not very manageable, as suggested by Leo in [9]. While it is easy to represent the extreme numerical cases, as soon as the combinations of one and two are considered in a classroom with students who are not familiar with combinatorial formulas and binomial coefficients, a process to represent the total of 89 possibilities is not easy to discern. It may occur that one possibility is counted twice or that some possibilities are missed during the counting. Nonetheless, there is a pattern embedded in the resolution whose exploration can be strategically approached by starting with staircases which have fewer steps (the 3-step and the 4-step staircases in [9–10]). Although the students from this group do not communicate a pattern, they examine reductions of the problem and, hence, approach the challenge of solving the problem without adding up the total number of ways of going down ten steps. They do so with the use of various languages during group work.

The everyday terms and colloquial grammars used by the students suggest a relaxed construction of the language to present mathematical thinking. In [1] and [8], for instance, Maria and Ton name the number words “one” and “two” in plural. In doing so, they communicate the possibility of grouping the numbers in a sequence regardless of their order of appearance. Moreover, in [9], Leo represents the quantity of eligible combinations for the resolution of the task as one that is “too much”; in doing so, he communicates a language of quantifying empirical observation. Seemingly, the use of language in this way facilitates the students to think of a shorter staircase. That is the beginning to produce a strategy to reduce the original problem to a simpler one. The language of mathematics in the wording of the problem somehow communicates approval to the use of a language linked to a particular empirical world. Other languages of mathematics with, for instance, an emphasis on algebraic grammars would have led to an experience of the everyday situation differently and possibly to visualize strategies not so much based on quantifying.

There are other moments in this data that do not directly involve the mathematical use of languages. In [2] and [4], Leo equates the movements of one step at a time with “baixar” (going down) and two steps at a time with “saltar” (jumping). He never uses these two words interchangeably. In [3] and [5], Maria and Ton communicate a meaning of “baixar” that also includes “saltar”. This is a moment of modeling how two words of the Catalan lexicon need to be understood. Moreover, the suggestion in [6] that Ada might interpret “baixar” similar to Leo can be seen as an allusion to the qualities attributed to the group of people that Leo and Ada are seemingly placed in. The intonation of Maria in [6] does not suggest a concern with including Ada in the discussion, but rather a concern with this students’ understanding of the words. Due to the broader context of official language policy and special lessons for latecomers described earlier in the chapter, we can interpret that references to shared backgrounds are suggested as well as the account of Leo, and possibly Ada, as poor users of the official language of instruction. All this interacts with the mathematical discussion in which the solving strategy introduced by Leo is followed up in the group work.

Example 2: “But she is interested in why eight” This second example occurs during the final part of the class discussion in interaction with the teacher (T). Language modeling is identified in relation to the right term in normative Catalan for steps (“escalons” in [4–5]). The explanation of the relationship between the solution of the problem and the solution for an 8-step staircase is communicated close to the replacement of the Spanish-rooted “escalons” [3] with “escalons.” In the critical discourse analysis that follows, a key assumption is that what is done by people with language is constructed by their understanding and interpretation of more or less local discourses.

<table>
<thead>
<tr>
<th></th>
<th>T</th>
<th>No heu acabat, però ho discutirem junts. Dificil? [You have not finished, but we will discuss it together. Difficult?]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Ton</td>
<td>Estem quasi a punt. [We are almost done.]</td>
</tr>
<tr>
<td>3</td>
<td>Maria</td>
<td>Un cop sapiguesm l’escala de vuit escalons, calculam la del problema. [Once we know the eight-step staircase, we calculate the one of the problem.]</td>
</tr>
<tr>
<td>4</td>
<td>T</td>
<td>Vuit escalons? [Eight steps?]</td>
</tr>
<tr>
<td>5</td>
<td>Maria</td>
<td>Volia dir escalons... [I meant steps.]</td>
</tr>
<tr>
<td>6</td>
<td>Leo</td>
<td>Pregunta el vuit. [She asks about eight.]</td>
</tr>
<tr>
<td>7</td>
<td>Maria</td>
<td>Vuit escalons és tresanta-quatre. [Eight steps is thirty-four.]</td>
</tr>
<tr>
<td>8</td>
<td>Leo</td>
<td>Però no interessa per què vuit. [But she is interested in why eight.]</td>
</tr>
<tr>
<td>9</td>
<td>T</td>
<td>Doncs si. Per què vuit? Per què no nou o set? [That’s right. Why eight? Why not nine or seven?]</td>
</tr>
<tr>
<td>10</td>
<td>Ton</td>
<td>Perquè quan tenim totes les maneres per vuit escalons, llavors els altres dos fins a deu es poden fer d’un en un o dos de cop. [Because when we have all the ways for eight steps, then the other two steps till ten can be done one or two at a time.]</td>
</tr>
</tbody>
</table>

This group of students discards the option of counting all the possibilities of going down the 10-step staircase one by one, and reduces the counting to a more manageable situation with the 8-step staircase. They communicate the partial answer of 34 in [7], and they claim in [10] that the calculation of the answer for the problem can be inferred. This is especially valuable due to the implicit mention by the teacher in [1] to the difficulties that this group may have experienced. This is not to say that this group is constructed as having more adverse circumstances than others. However, the evocation of unfinished work and difficulties is made in relation to a group in the classroom with two students who are still designated by the teacher as “latecomers”. From what is later said in the interaction, it cannot be inferred that the students produce the recursive pattern embedded in the resolution. I.e., the answer for each staircase is the sum of the answers for the two staircases that go before, given the decreasing order of number of steps. Nonetheless, their pattern implies counting up to 34 possibilities and thus it is not the most optimal, this is the only group that showed evidence of having produced a pattern in their search for strategies for not counting up to 89.
In this piece of language data—similar to what happens in Example 1—, the everyday terms and colloquial grammars used by all participants suggest a relaxed construction of the language to voice mathematical thoughts. In [6], for instance, Leo names the number word “eight” without communicating, therefore, its situated use as a measure. The teacher in [9] signifies number words as quantities instead of measures as well; in this way, she implicitly communicates approval. The use of formal languages of mathematics would have led to note that a number by itself is not a quantitative observation that can be accepted if it is not connected to a precise metric. Indeed, there is an interesting alternation of numbers by themselves and nouns with numbers in some of the turns so that in some occasions the students and the teacher communicate their attention to the empirical world suggested by the wording of the problem. Nonetheless, such alternation in this context does not primarily communicate precision of the metric, but rather precision of the noun in Catalan.

The moments in this data that do not directly involve the languages of mathematics are also interesting. When the teacher produces a question related to what Maria has said, the word “escalons” is replaced by “escalons”. This is not a semantically motivated distinction like the one with going down and jumping in the previous example. Maria, by first using a Spanish root term not found in the Catalan lexicon, makes a contrast with common language use in ‘her’ community of Catalan-dominant speakers. An intention of correcting vocabulary cannot simply be inferred, but such correction is communicated. It may well be that the teacher responds in a way that primarily intends to promote the explanation of the strategy by the students and, in so doing, she refers to “steps” with the word she is more used to. In [9], it seems that the teacher wants to make sure than an explanation of “why eight” and “why not nine or seven” is developed. Maria responds to what she may understand as a correction of a word, whereas Leo goes to the mathematics involved in considering the case of the staircase with eight steps. In this way, together with the questions in [9], the emphases placed on “eight” and “why eight” are fundamental in the construction of explanations in the middle of discourses of linguistic normativity and of numerical results. An idea of recursivity is produced, along with issues of generalizability of a mathematical solution related to an 8-step staircase.

5 Language and Multilingual Mathematics Learning

Along with exploring how participants in a classroom discursively construct some mathematics learning opportunities, I have explored how they construct these opportunities throughout the communication of concerns related to the recognition of language systems and language capitals in the previous section. The identification of a diversity of languages within language has been crucial in the understanding of the complexity of producing and communicating mathematics learning opportunities.

From the analysis, it follows that the construction of mathematics learning opportunities rests not only upon the possibility of producing and communicating mathematical ideas, but also upon the possibility of connecting the discussion of these ideas to the discussion of linguistic and discursive features of the students’ languages. Rather than contradictions, what we have are purposes that necessarily implicate the other. Moreover, the analysis indicates that language cannot be viewed as an actual resource for mathematics learning independently of its use in discourse. Either way, all this offers an empirical approach to language as resource that connects the development of mathematics learning opportunities to language. Elsewhere (Planas 2014; Planas and Setati-Phakeng 2014), other empirical approaches have been developed with extracts of data from research contexts in South Africa and Catalonia. Nonetheless, on those occasions, the potentiality embedded in the consideration of language as resource was not examined. What participants in the mathematics classroom do with languages should not be examined aside from what they can do.

Beyond the empirical approach to the view of language as resource, consistently with the adopted notion of language as discursive construction, we come to the potentiality of language for developing discourses that primarily produce and communicate mathematics learning opportunities. Given the fact that language is endogenous to the discursive co-construction of language systems and language capitals, this potentiality can be identified throughout the communication of mathematical ideas in a variety of social languages. Potentiality refers to the more or less incompletely developed mathematical ideas embedded in any situation of language use. The communication of ideas that would facilitate some processes of mathematics learning in a classroom is never complete in an ideal manner. There are always more meanings to identify and communicate and further possibilities to be accomplished.

The quality of potentiality, therefore, distinguishes what can be realized from what is actually realized. Language is always developing and so is the realization of its potentiality for resourcing the communication of mathematical ideas. Thus, it is not only what is done that makes language a resource but what can be done; and it is precisely what can be done that enables to critically think of multilingual mathematics teaching and learning in terms of becoming something different. In the data exemplified, the students use their languages to develop solving strategies. Such learning opportunities are produced due to what these students do with language. But even if those opportunities had not been produced, they would be there, as potential outcomes, latent in and dependent, at least, on the co-construction of language systems and language capitals in discourse.

6 Final Discussion and Some Implications for Teaching

I have tried to illustrate the complex relationship in discourse between the construction of mathematics learning opportunities and the construction of levels of meaning concerned with the recognition of proper languages and proper students. At this point of our knowledge of language in mathematics teaching and learning, one distinctive point is whether some levels of meaning may be intentionally produced and, to some extent, may be produced separately from some other levels—in particular periods of time in a given context of culture. That is as much as asking whether
language can be made to work mostly in teaching for the purpose of mathematics learning. In Planas (2014), I maintained the importance of modeling the culture of the mathematics classroom for flexible uses of language to arise and be valued in the interaction. One of the conclusions of that work was that practices like code-switching needed to be explicitly produced in discourse. My most current research tells us something about what else needs to be produced, namely, the affirmation of a diversity of languages in mathematics teaching and learning. From the perspective of the so-called languages of mathematics, considerate reflection is required in order to challenge the ideological underpinnings that present some of them as natural. This calls for explicit attention to language in teaching.

The next transcript with Ada and the teacher below offers some hints about how to produce and communicate a kind of critical language awareness in teaching:

<table>
<thead>
<tr>
<th>Ada</th>
<th>Després de dos números. [After two numbers.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher</td>
<td>Doncs ara necessites una altra frase. Després de dos números, qué pasa? [So you need another sentence now. After two numbers, what happens?]</td>
</tr>
<tr>
<td>Ada</td>
<td>El tercer. [The third.]</td>
</tr>
<tr>
<td>Teacher</td>
<td>D'acord, però això no és una frase. Què li passa al tercer? Després de dos números, el tercer hi és? Cada dos nombres, qué pasa? [Okay, but this is not a sentence. What happens to the third? Every two numbers, what happens?]</td>
</tr>
</tbody>
</table>

Consistently with the discussion of the potentiality of language as resource for mathematics learning, the question of how teachers and students can use language for mathematics teaching and learning is essential. Theoretical and empirical work need to include the discussion of possibilities along with interpretations of what was actually done with language. A dominant discourse in mathematics education research is precisely the construction of findings regarding what participants in a study do, how they do it, and why they do it in that way. Such discourse in the field needs to be widened with questions about what participants could have done, and why other possibilities were accomplished instead. By interrogating data and findings with these additional questions, we can raise the level of understanding of data and the theoretical reach of the theories, analyses, and findings undertaken in our investigations. This particularly applies to studies of multilingual mathematics teaching and learning. It is important to assume that in all situations of communication language entails the construction of both difficulties and opportunities. Such assumption could avoid biases toward explaining extreme stories of either success or failure. By putting the value not only in what participants and researchers do with language, but also in the possibilities language enables, discourses of practice and innovation in mathematics education might be widened as well.

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Differential Enhancement in Mathematical Pre-School Class Activities

Ola Helenius, Maria L. Johansson, Troels Lange, Tamsin Meaney, and Anna Wernberg

Abstract In this chapter, an adapted version of Dowling’s distributive strategies is used to show how two children, Klara and Teo, are provided with differential enhancement in the mathematical learning opportunities that they are offered. The analysis shows that the use of everyday settings of mathematics problems, including expectations about the social relationships in those settings, can cause children to collude in the kind of enhancement that they experience. Expectations about the social relationships, within the problems being solved and between the participants, contributed to the two children using strategies which channelled them towards operating in different domains.

Keywords Recontextualisation · Strategies of distribution · Micro-ethnography · Pedagogic strategies · Opportunity

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