In January 2005, the two authors and eight secondary mathematics teachers started working together under the name of ‘Critical Mathematics Education Group’, having been inspired by the principles of the critical social theory as interpreted by Skovsmose (1994)\(^1\). Since then Núria coordinates this group, while Marta serves as a consultant based on her prior work with a group of teachers with similar goals in Tucson, Arizona, U.S. We claim that what is said and how it is said in the classroom may transform mathematics education into a form of critical mathematics education that promotes the teachers’ and students’ empowerment or, conversely, may increase the experience of learning conflicts (Alrø & Skovsmose, 2004). For this reason, we are especially interested in studying classroom discourse. Elsewhere (Planas & Civil, 2009) we have summarised the work with the teachers of the Group in this project.

The interest for the study of discourse processes in the mathematics classroom and its effects on the learning has increased in recent years. Some studies (Skovsmose & Penteado, 2009; Vilela, 2007) point out the importance of analysing the diversity of meanings emerging from social interactions in the mathematics classroom as well as the implications of not negotiating these meanings. Learners are defined as people who need to come to some sort of agreement in order to communicate while doing mathematics. In our work within the ‘Critical Mathematics Education Group’, we draw on a critical (Skovsmose, 2005) and socio-cultural approach (Stephan, Cobb & Gravemeijer, 2003) to education that sees the classroom as a social structure in which participants develop strategies that help maintain certain positions and reduce others.

In this chapter we explore the influence of discourse processes on the interpretation of norms in part of a mathematics lesson in a working-class school in Barcelona, Spain. The set of data that we present uncovers some of the discourse processes that are expected to reproduce conflicts, while the final section reflects on possibilities of

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transformation. During his stay in 2004 as a visiting scholar at the University where Núria works, Ole read the classroom transcript of this lesson and offered some very useful insights that we incorporated in the analysis. The week before he had visited the school, and had talked with some of the students and the teachers. He later mentioned the visit and the lesson in his lecture “Globalisation, ghettoising, and uncertainty: Challenges for critical mathematics education”, at the International Congress on Mathematics Education in Copenhagen (Skovsmose, 2004).

CLASSROOM NORMS

Norms can be defined as accustomed collective ways of perceiving, believing, evaluating and acting in a context (Goodnough, 1971). They refer to both the culture of a group of individuals—those shaping a mathematics classroom, for instance—and to the identities of each individual. The interpretation of one norm gives information about who is interpreting it. For instance, when a teacher systematically rejects the group work dynamics, s/he is enacting a specific identity as a particular kind of teacher. In the classroom context, norms contribute to structure the practice. Teachers communicate their expectations through the development and enactment of norms. Norms have then to do with regularities and the expected forms of knowledge and behaviour. Although they do not need to be jointly established by the teacher and students, there is a need for a shared understanding. Teachers can make their expectations known to students by making explicit the classroom norms and providing opportunities for the students to practice them even if these norms are not consistent with their own expectations. Chronaki (2005) focuses on the experience of minority students when learning school mathematics. She argues that the multicultural mathematics classroom can be a place where minority students’ experience difficulties when adjusting to norms that differ from the norms in the other communities in which they are involved.

Our definition of norm includes the idea of change and reconstruction. Since norms are developed and maintained through the interactions of individuals, they can change themselves and make the classroom culture change. At a collective level, norms allow for the reconstruction of themes between teachers and students and amongst students engaged in classroom interactions. At an individual level, discrepancies between what students expect from a classroom and the established norms may lead them to modify part of their initial interpretations (Planas, 2007). On the other hand, the contexts in which the norms are established can also change by implementing new norms. Yackel and Cobb (1996) point to the importance of the social interactions in the development of norms and the modification of practices. Sullivan and Mousley (2001) talk about norms in relation to routines and modes of communication that impact on the approaches to learning that teachers choose, the responses they value, their views about legitimacy of knowledge, the responsibility of learners, and their acceptance of risk-taking and errors.

In Planas and Civil (2002) we argue the problematic nature of the processes of learning norms. Classroom norms are rarely openly discussed though they are the practices governing interaction in a group. Teachers may suggest that an answer for
a mathematics problem does not have to be final, and that continuing to revisit it is a useful practice. They may also suggest that there may be more than one answer. However, these norms are established in relation to specific learning situations, and students may not know whether to apply them in other situations and how to do it. In this case, students need to learn to expect that there may be more than one answer (so that they place more value on finding different methods for solving the same problem) and that answers must be revised in any given situation.

CLASSROOM DISCOURSES

Gee (1999) characterises discourse processes through the notion of D/discourse. Discourse, with capital D, refers to a socially accepted association between forms of using language, thinking, believing, valuing and acting that can be used to identify those who belong to the socially significant group and to point out the role of each member of the group. The Discourse of a mathematics classroom is then the set of legitimate norms that shape the classroom culture: what is accepted as a mathematical proof, what criteria are considered in the process of solving a problem, what is the role of the teacher, or who is in charge of establishing conclusions. The discourse consists of all the co-existing forms of using language in action, including those that are not expected.

Gutiérrez, Rymes and Larson (1995) make a distinction between the Discourse of an object to be studied, such as the Discourse of Mathematics, and the discourse processes of a context, such as the discourses of a classroom. By looking at discourse processes, we may see processes of social and individual change when interpreting classroom norms that would remain invisible if we only focused on the mathematical Discourse as a final and already established product that needs to be acquired. The dynamic notion of discourse processes (Graesser, Gernsbacher & Goldman, 2003) does not confer such strict rules as Discourse because the established norms are not seen as ‘undoubtedly legitimate’ but as ‘discursively legitimate’. On the other hand, the study of discourse as a process involves looking at how social processes intervene in the construction of social relationships and positions within a particular context. The focus on social processes requires an understanding of the micro and macro contexts that have an influence on them. One problem is how to simplify the analysis of contexts and the interactions among them without losing crucial information in order to understand the role of specific discourse processes when only having data concerning the acts of face-to-face communication.

Classroom D/discourses provide a window into the mathematical learning processes. They show the ways of thinking that are valued and how these ways of thinking change. One particular Discourse is constructed on the basis of different discourses with specific values. The influence of other discourses would lead to a different Discourse and would probably put a higher value on different knowledge. A classroom discourse may have ‘collaboration’ as a value and the students’ desks arranged in a circle as a symbol. The learner of mathematics needs to show competence on this discourse. If s/he does not work together with her/his peers,
the teacher (and the other students) might think there is some kind of mathematical shortcoming with this student. Sfard and Kieran (2001) have talked about learning in terms of becoming a participant in various discourses. Knowing the Discourse of Mathematics and the discourses of the classroom means knowing the legitimate norms that regulate participation, as well as the acceptable content and form of the various discourses prevalent in the broader culture. Learning mathematics is then equated to the process of adjusting to certain norms that have not always been well defined in the class.

THE MICRO-CONTEXT

The data we present were collected in a mathematics classroom with sixteen 15-16 year old students. The class is located in a high school in a working-class area of Barcelona where contact between groups of different ethnic backgrounds is frequent. Núria had been a teacher in this school for three years, several years prior to this study. Most students work in low-paying jobs in the afternoons and during the week-ends. In the class there are four second-generation immigrant students who have a good competence in the two official languages —Catalan and Spanish—, though they tend to use Spanish. The other students are local Spanish speakers who can speak Catalan, which is the language with a higher status in the school context (since 1983 the official language in the educational system in Catalonia is Catalan).

The school had been classified as ‘special needs’ by the local Administration based on the high number of students from low-income families. In order to create balanced ‘heterogeneous’ classes, students are distributed according to criteria of gender, ethnic background, perceived ‘academic ability’ and ‘learning disabilities’, as well as different behaviour issues established by the group of teachers. Students are not told into which categories they are placed, although references to their performance are usual. Catalan special needs high schools have at most 20 students per class —regular high schools have an average of 30 students. The curriculum at the two types of high school is also different. While in the regular high school students are in an academic track towards the external standardised exam to enter the University, in the special needs schools, students are not expected to go to University and teachers do not have the pressure to practice certain exercises. The method of instruction is also different. In regular schools, most lessons are lecture-based with reading and homework assignments, while in special schools many teachers encourage collaborative learning environments.

The teacher, with fifteen years of teaching experience, four of them at this school, is a local Catalan speaker. He uses Catalan to give instructions and occasionally switches to Spanish. We selected a teacher who promoted whole class discussions, whom had his students’ desks arranged in groups, and who was a member of our Group. Although his teaching is traditional in many respects (students are asked to write everything down in their notebook, the blackboard is often used...), there are some reform-oriented aspects, such as students working in
small groups engaging in discussions of problem-solving tasks, with the emphasis on the process of solving mathematical problems and not so much on searching for concrete results.

The students worked in their small group on the following problem.

Here you have the population and area of two neighbourhoods in your town.

Neighbourhood 1 (A)  Neighbourhood 2 (B)
65 075 inhabitants 190 030 inhabitants
7 km² 5 km²

(i) Discuss in which of these places people live more spaciously.
(ii) Find how many people should move from one neighbourhood to the other in order to live in both of them spaciously.

The actual names of the neighbourhoods were given to the students. A stands for a well-known neighbourhood in a wealthy part of Barcelona where Catalan is the prevalent spoken language, and B stands for the school neighbourhood. This was intentional to make the problem more real to the students. Ole’s work has been especially committed with this type of semi real problems in the development of teaching and learning classroom projects (see, for instance, Alrø & Skovsmose, 2004, for the description of projects). The year before, the students had worked on proportionality and had studied equations. Thus, they were “supposed” to have the mathematical skills required to solve the task.

The 12 minutes episode we discuss here starts at the beginning of a whole group discussion, after having had some time for working in small groups. One of the students, Laura, tells the teacher that her group has not developed a common approach to the problem. This is the main reason for choosing this group. The episode centres on three students —Laura (L), Emilio (E) and Mateo (M)— and the teacher (T). They all are local students but the two boys, Emilio and Mateo, are South-American second-generation immigrant students with a good knowledge of Catalan.

EXAMPLES OF CLASSROOM DATA

Our assumption is that norms of the mathematical practice that are being differently interpreted by different participants may lead to conflicts. Hence, we focus on participants’ interventions that we consider to be examples of such conflicts.

The Initial Scenario

We present three excerpts from the communication of this group’s work on the problem. The teacher and Laura use the target language, Catalan, and do not switch to Spanish during this time. Emilio and Mateo use Spanish and do not switch to Catalan. In Excerpt 1 a conflict occurs when the teacher tells Emilio that the
reasoning used when answering the first question of the problem will not be useful when trying to answer the second question ([8]). Another conflict occurs when the teacher does not accept Emilio's proposal to change the statement of the problem in order to make it more meaningful and the issue of not using mathematics or not being mathematical arises ([15], [16]).

Excerpt 1

1 L: This is a problem about densities because data are about densities.
2 T: Okay. Tell Laura that she needs to explain herself better. [To Laura] We know that you know a lot, but...
3 L: In A [the name of the neighbourhood] the density is lower than in B [the name of the neighbourhood]. That's all.
4 T: Emilio says no.
5 E: I just don't see it! There's something missing.
6 T: [To Emilio] How have you done it?
7 E: It's clear that here [B] there are more people and less space. I've been there. Flats are very small.
8 T: Okay. You say it's clear, but then how do you answer the second question? You cannot say again that flats are very small.
9 E: The second question is wrong.
10 T: Why?
11 E: I wouldn't move alone, I'd take all my family.
12 T: What do you mean?
13 E: I would change the second question.
14 T: Don't start again, Emilio! You know that problems are like they are.
15 M: I don't mind changing the question, but if you change it, we won't practice the mathematics that the teacher wants us to practice. You can do it by trial and error; you first try with 50 000 people.
16 L: That's not mathematical!

In Excerpt 2 Laura is at the blackboard explaining her approach to the whole class. She pays attention to the mathematical procedure and reduces the task to a series of properties of equivalence. She makes the mathematical context prevail over the real context by considering the problem as part of a group of problems about densities (see turn 1 in Excerpt 1), and not including comments on the real context (that Emilio had brought in [7], [11]).
Excerpt 2

23 \(L:\) [On the blackboard]
\[
\frac{65075}{7} \rightarrow \frac{65072}{7} = 9296 \text{ h/km}^2 \text{ at } A; \\
\frac{190030}{5} = 38006 \text{ h/km}^2 \text{ at } B; 9296 < 38006
\]

24 \(T:\) Okay. We need to compare the two neighbourhoods. These numbers mean nothing if we don’t compare them.

25 \(L:\) This number [9296] is...

26 \(E:\) We place some people here and some people there.

27 \(L:\) Let me finish! 9296 is smaller than this number [38006]. This means that here [she points to \(A\)] you live more spacially.

28 \(T:\) Okay.

29 \(L:\) Now let’s see the equation. [On the blackboard]
\[
\frac{190,030 - x}{5} = \frac{65,072 + x}{7} \quad \frac{38,006 - x}{5} = 9,296 + \frac{x}{7} \quad 38,006 - 9,296 = \frac{x}{5} + \frac{x}{7}
\]
\[
28,710 = \frac{12x}{35} \quad x = \frac{28,710 \cdot 35}{12} \quad x = 83,737.5 \rightarrow 83,737 \text{ people}
\]

30 \(T:\) Laura, you have to explain what you’ve done and why.

31 \(E:\) I don’t understand why she replaces 65,075 by 65,072.

32 \(T:\) Laura? Why do you replace it now?

33 \(L:\) [Back in her place] I’ve already explained my approach, now they explain theirs.

Excerpt 3 shows how Emilio and Mateo focus on the real situation suggested by the statement of the problem. They do not consider it necessary to make the mathematical context prevail over the real context. For them, the task is concerned with understanding aspects of a real situation rather than just with mathematical calculations being applied to that situation ([41]). From the beginning of the task, they base their approach on their ‘out-of-school’ knowledge. For Emilio, his knowledge of these neighbourhoods prompts him to say, “I’ve been there. Flats are very small”. During the group work (which is not part of the transcript we discuss), Mateo had made similar comments showing him connecting the problem to his knowledge of the neighbourhoods, while Laura does not publicly refer to her ‘out-of-school’ knowledge.
Excerpt 3

38  T:  Mateo, let's concentrate; now you forget about the people and you only think about the fraction. Is 65075 a multiple of 7?
39  M:  No.
40  T:  That's the point! 65072 is a multiple of 7 and 65075 isn't. Now it can be divided exactly.
41  M:  But it's not about multiples, it's about people!
42  E:  In the last operation, she doesn't look for multiples, does she?
43  L:  This is not important.
44  T:  Do you see how she has solved the equation?
45  M:  Yes.
46  T:  This is important.
47  M:  Could we just give an approximate solution?
48  L:  Please, this is not important.
49  M:  Shall we copy the equation?

The differences in the students' interpretations of the task lead to differences in the interpretations of norms that regulate the practice of solving ‘contextualised’ mathematical problems. Mateo and Emilio focus on the real context of the problem as informing their approach to solving it, while Laura focuses on the mathematical procedures to solve the problem, leaving aside any connections to the real context.

We first discuss what we have labelled as the confronting scenario, where the interpretations given to norms about ‘contextualised’ mathematical problems are confronted. Second, we look at the reconstructed scenario, where changes in some of the interpretations occur. The confronting scenario is especially relevant in the data we discuss here. It is presented as a place where the initial interpretations come into contact and are confronted, some of them becoming part of Discourse in the reconstructed scenario, while some others not being referred to again.

The Confronting Scenario

The teacher expresses a different comfort level with the contributions of the different students and confirms Laura’s approach in several turns ([2], [28], [44], [46]). When Laura changes 65 075 to 65 072 on the blackboard, she is removing 3 people from the original number of people because she wants to work with a multiple of 7. This can also be seen as an example of referring to reality because it is meaningless to talk about 3/7 of a person; however, she does not talk about the importance of the real context in her resolution, neither does she react to Mateo when it turn [41] he says, “But it's not about multiples, it's about people!” Her use of an equation when answering the second question is yet another way to seek mathematical rigor, while from a real life point of view, this level of exactitude is not needed. She regards mathematical rigor as a more important characteristic than
one based on real life, and she makes sure to use proper symbolic and oral communication when doing mathematics. Conversely, the teacher rejects Emilio's approach by saying that the reasoning he is using when answering the first question will not be useful when trying to answer the second. Instead of developing a 'more mathematical' reasoning or a reasoning less based on his 'out-of-school' knowledge, Emilio first holds on to his argument and Mateo partially joins him.

When Emilio says, "I just don't see it! There's something missing", and continues with his comment on his knowledge about $B$, we wonder if he is questioning what the problem is asking, since to him, that first question seems obvious, given his knowledge of the neighbourhoods. The teacher shows interest in Emilio's approach although his reaction, when he tries to get him to think about the second question in the problem, is somewhat dismissive. As the episode goes on, the teacher continues to dismiss Emilio's interventions (such as his wanting to change the second question) and turns to Laura to move on with the problem. Then the teacher and Laura display their emphasis on the instrumental nature of the task, stressing the mathematical procedures and not taking into account previous comments concerning the importance of the context.

Emilio and Mateo have difficulties completing their approaches. They struggle with trying to understand Laura's practices such as comparing two ratios though the use of the ratios has not been justified. They spend more time trying to clarify Laura's strategy than insisting on their own strategy. Emilio begins by trying to make sense of Laura's approach, then he centres on his ideas and, at the end, he is again committed to making sense of his peer's approach. Emilio and Mateo seek a relationship between the mathematical and the real contexts that arise from the problem. They attempt to introduce a 'real context' approach to the solving process, and also contribute to maintaining Laura's procedural approach by repeatedly asking questions about it. Emilio wants to know about 65 075 being replaced by 65 072 ([31]). They are highly involved in the discussion about the task and are often willing to share their reasoning. Their efforts contrast with Laura's who only partially exposes her ideas and mostly reduces her explanations to the written register on the blackboard. When Emilio asks for more explanations, Laura avoids giving an answer and the teacher does not clarify the situation either. The teacher does not encourage other students to refute Laura's ideas, nor does he ask her to refute the trial and error approach or the use of real context.

When the teacher concludes, he bases his comparison of neighbourhoods on comparing the densities. He points out that the ratios given by the calculation of the two densities do not lead to relevant information in the problem context unless they are compared with each other. Laura does exactly that, to the teacher's approval, while Emilio's attempt to follow a trial and error strategy is ignored. Even when Mateo brings it up again, the teacher brings them back to Laura's proposal. There is no discussion about what it means to compare densities or whether data from the problem should be compared without calculating densities.

The teacher's attitude is similar to Laura's. He adopts an inquiring attitude, but he does not offer explanations and does not always wait for answers. We cannot know whether Laura and the teacher understand Emilio's and Mateo's ideas well.
but, in any case, they do not try to elucidate them. When Emilio asks for explanations about $65\,075$ being replaced by $65\,072$, the teacher does not answer but instead he changes the focus. Later, he refers to the task as an arrangement of rules. The division of $65\,075$ by $7$ is interpreted as a symbolic task with no real meaning linked to the statement of the problem. Mateo does not understand why they should worry about numbers being divided exactly by other numbers. Emilio does not understand either why Laura considers multiples in one of the divisions but then, when solving the equation, does another division problem without looking at multiples, obtains a decimal number and then rounds down the result. There is no clarification for Laura’s practice. When dividing $65\,075$ by $7$, it is easier first to round down, ‘eliminate’ $3$ people and then to divide; the teacher shows his approval of this method. Later, Laura rounds down $83\,737.5$ after having divided. It is not clear if she rounds down because she is aware that talking about half a person does not make sense. While Emilio is trying to understand what she is doing, Laura and the teacher seem to dismiss him.

The teacher presupposes Laura’s knowledge but he does not make a similar comment when addressing the two boys. However, during the small group discussion, Emilio and Mateo work together and show a deep understanding of the trial and error strategy. With the second question in the problem, they begin by checking what happens if $50\,000$ people move out from one neighbourhood to the other, then they check with $100\,000$ and, finally, they check with $80\,000$ people, which is very near to the ‘exact’ response obtained when applying the equation. This is reflected in the copies of the students’ work. They may not have a need to use the equation because they find it reasonable to give an estimated answer that can be supported by knowledge from their life experiences. This knowledge makes it rather immediate to answer the first question in the problem. Although it is true that by just looking at the given numbers, one can conclude that $A$ is more spacious, it may not be so obvious since we do not know how big the flats are.

The Reconstructed Scenario

When Ole read the transcript of this episode, he emphasised how for the two students —Mateo and Emilio— who tried to follow a classroom norm about referring to reality when solving ‘contextualised’ mathematical problems, the teacher just seemed to change the norms so that the students would feel excluded. This is a good summary of the reconstructed scenario that emerges from what happened in part of the lesson that we have presented. The fact that the teacher posed a ‘contextualised’ task stimulated practices that are open to explore situated mathematics. Mateo and Emilio were well aware of ‘the norm of contextualisation’, and they explicitly referred to it when they found that their knowledge of a neighbourhood was relevant to the resolution of the problem. But the teacher, who had first supported the idea of situated learning in mathematics, abandoned ‘the norm of contextualisation’, especially when claiming that the key point of the resolution was about multiples.
Emilio and Mateo modify their initial interpretations concerning how to deal with the resolution of ‘contextualised’ mathematical problems. The teacher’s and Laura’s interpretations become part of the Discourse, while Emilio’s and Mateo’s interpretations are obstructed by these students’ practices having been evaluated as non-mathematical. At the end, the instrumental nature of the task has been reinforced by the acceptance of those who had questioned it, and the references to the real context and the trial and error strategy have been rejected. Emilio and Mateo accept copying the equation in their notebooks and give up on their attempts to change the approach to the problem for the rest of the session. They write down what Laura has done and stop arguing their point. Emilio writes the teacher’s words in his notebook, even though a few minutes earlier, he had inquired about the use of densities and had not received any response.

The teacher centres his efforts on the process of institutionalising Laura’s approach to the problem. His discourse is based on the validation of mathematical procedures that adjust to a somehow changing interpretation of ‘the norm of contextualisation’. He does not show interest either in the students’ ‘out-of-school’ knowledge concerning the two neighbourhoods, or in how they organise their strategies by using this knowledge. Emilio points out some practical ideas that are likely to be effective in a real situation, but his contribution is rejected. While adding to Laura’s ideas, Emilio and Mateo give up looking for clarification of the mathematics involved in the episode, although they may have not been convinced of Laura’s approach.

The disparities in the interpretations of norms lead to learning conflicts. A conflict has to do with how to determine the valid methods of solving ‘contextualised’ mathematical problems when they seem to be different whether being applied in a real life context or in the institutional context of the mathematics classroom. Emilio insists on using his ‘out-of-school’ knowledge and shows a certain resistance towards the teacher’s interpretations. By using his ‘out-of-school’ knowledge, he resorts to his understanding of an appropriate way of solving ‘contextualised’ problems in the mathematics classroom. He insists on interpreting the solving process in the real context that is suggested by the statement of the problem. Mateo also refuses to focus his approach on the fractions and its representation on the blackboard. When other groups intervene, the issue of the ‘out-of-school’ knowledge arises again and the teacher suggests not reproducing the former discussion.

THE MACRO-CONTEXT

Our data suggest that classroom discourses lead to differences by considering the students’ interpretations of the norms and valuing the norms and the students differently. Laura and the teacher justify their actions by rejecting Emilio’s and Mateo’s proposals on behalf of the mathematical knowledge, and do not facilitate these students’ participation. We cannot conclude, however, that these differences are due to something that has happened within the
classroom. They may be already considered differently at the time they enter the classroom due, for instance, to their gender, ethnic, and language identities. On the other hand, although they are not used to solve ‘contextualised’ problems, they may have developed a common understanding of classroom norms concerning former learning experiences that should be enough for them to facilitate dialogue. It would be interesting to consider what factors would be influential in making a group more likely to modify their working relationships.

Despite the fact that these students are working-class, class differences are still expected to influence the social interactions and the reconstruction of classroom norms. The types of differences that arise between Mateo and Emilio on the one hand and their teacher and Laura on the other, are classic class differences, as described by Cooper and Dunne (2000), who found that working-class students tend to take contexts more seriously than middle-class problem creators intend. Lubienski (2004) argued that working-class students tend to become immersed in real-world constraints of problems, and are also more deferential to the authority of the teacher. This fits with our data about Mateo and Emilio being concerned about the teacher’s opinion of them.

The experience of conflict could have led to an opening up of social interactions that served to clarify the norms. However, participants focus on the differences instead of making an effort to overcome the conflict. First, the conflict reveals the limitations in the school mathematics repertoire of the students: Emilio and Mateo do not succeed in de-contextualising the problem, and Laura does not succeed in contextualising it at the beginning of her resolution. Second, the conflict also reveals the limitations in the negotiation of norms of the mathematical practice. Participants are not having discussions about the interpretations on which they disagree nor are they trying to reach partial agreements. While one participant proposes to change an interpretation, another participant acts as if that change was not worth considering and proposes, in turn, a change or an agreement concerning a different interpretation. It could be that the teacher has trouble managing and facilitating this type of discussions. But even so, the participants bring up very different problems (the inadequacy of some observations, the timing of the session, the need to conclude, the general lack of meaning of the mathematical task) without jointly deciding which to deal with first.

The final resolution of the disparities concerning the use of the real context when solving a ‘contextualised’ mathematical problem, as well as the teacher’s and Laura’s attitudes, may have led Emilio and Mateo to think that their approach to the problem is wrong. From a mathematical point of view, however, these students’ alternative proposals are right. The application of the method of trial and error is correctly developed in the students’ notebooks. When these students publicly revise their initial interpretations to norms and change them, they may be accepting as wrong mathematical practices that are correct. If this is the case, the resolution of the disparities is itself an obstacle to the learning of mathematics in Emilio’s and Mateo’s classroom because other students may be getting the same message.
Skovsmose (1994) gives a conceptualisation of conflict as a necessary form of human action that occurs in complex social settings. This leads to the understanding of conflict in the mathematics classroom as social continuity. It assumes that conflicts are the result of the longer-term discursive processes that have established conflict as a form of institution linked to social practices. A critical interpretation of mathematics education must incorporate the understanding of conflict and the efforts towards transformation. Our work seeks to uncover the nature of classroom discourses and their relationships to the institutionalisation of mathematical practices. We argue that an awareness of the existence of conflict as being manifested in the classroom is an essential part of any empowerment process.

Empowerment requires paying attention to the limitations but at the same time formulating new problems in which it makes sense to take responsibility for actions. In our work within the 'Critical Mathematics Education Group' we claim the necessity of addressing the processes of empowerment of students and teachers as being inseparable parts of a same reality. To understand the mathematics underachievement of certain groups, we need to understand the perceptions of opportunities to learn, from the students' and the teachers’ perspectives. In particular, the teachers’ empowerment needs to start with a reformulation of reality in terms of opportunities that permit them to view conflicts as a legitimate part of the learning process.

We work with the teachers to help them develop the ability to work with other teachers and with their students in order to change expectations. When being presented the data introduced in this chapter, many teachers of the Group expressed negative evaluations of the practice of the teacher in the episode. In support of their negative evaluations, they mentioned the fact that the teacher had not handled some of the interactions with the students well, or that he had created a negative learning atmosphere for some of them; they also suggested how the teacher in the episode should have behaved. Most teachers reacted by projecting their own models concerning the teaching and learning of mathematics, as well as their perceptions concerning what a mathematics teacher should be. They evaluated the practice of the other teacher by reflecting on their own practices and idealising them.

During the time for group's discussion on classroom data, the teachers in our Group succeed in creating an environment where their perceptions are opened for questioning. There are moments when new practices may be controversial because they disrupt ways of thinking but, even in these situations, the proposal of change appears as empowering because teachers accept talking about the students’ opportunities and their own opportunities when facing conflicts. Collective empowerment, however, does not mean linear and smooth progress towards a shared feeling of inclusion and mastery, although most teachers in our Group are responding positively. By gaining empowerment, conflicts are not necessarily overcome and old practices are not totally
eliminated. However what is new is that teachers now can see situations of conflict as potentially creative opportunities for transformation and empowerment in learning.

NOTES

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Núria Planas
Universitat Autònoma de Barcelona
Spain

Marta Civil
University of Arizona
USA