1 Introduction

Economic entities are becoming more and more interconnected with each other and the overall degree of international equity market connectedness has been increasing especially over the past two decades. Although, there is an evidence that there is a strong connection among markets, new approaches of measuring and quantifying the connection and understanding its dynamics are still being developed.

We measure the connectedness using the stock market data because of the advantages to analyze the economic links between two countries on the basis of stock market data, rather than aggregate economic data published by national statistical offices, is that stock market data are readily available, allowing analysis in almost real time.

To measuring the connectedness, I have three different purposes in my PhD thesis. First, we measure the connectedness of different stock markets using return spillovers by the methodology of Diebold and Yilmaz [2]. My contribution in this case, will be not only looking on the return spillovers but also develop a methodology to measure the connectedness by using volatility-to-volatility spillovers. The details of the method will be explained in the following sections of this paper. But for now, we need to emphasize that to compute the volatility spillovers we need the weekly volatility series of each market. To do this, we develop a methodology to obtain the weekly volatility series using the German and Klass [14] methodology. Below, we will explain the objective of this methodology and the intuition behind it.

One of the approach to measure equity market connectedness, which we mentioned above, is based on forecast error variance decomposition, developed by Diebold and Yilmaz [2], and extensions of this approach to assess the propagation of information across markets, developed by Schmidbauer, Rösch and Uluceviz [8].

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One way to quantify market connectedness is to estimate the variance of the error when forecasting future return or return volatilities on the asset price. Diebold and Yilmaz [2] [3] use this idea to develop a network view of markets as nodes and weights determined by variance shares. They decompose the error variances of joint asset return forecasts, using the vector autoregressive (VAR) models. In terms of their approach, the decomposed error variances present a network with assets as nodes and the weights of links between nodes determined by shares of forecast error variance spillovers. Using this methodology, pairwise spillovers can be discussed, but their goal is to go one step further and designate the degree of market connectedness. They suggest a measure for the degree of market connectedness which they call the spillover index.

Schmidbauer, Rösch and Uluceviz [8] extend the spillover index methodology and quantify dynamically the amount of connectedness of markets with a focus on the flow of information. They use the concepts of Kullback-Leibler divergence developed by Demetrius [1] and Tuljapurkar [13] to measure the amount of market connectedness dynamically. Schmidbauer, Rösch and Uluceviz [8] discuss the Kullback-Leibler divergence in the field of market connectedness and define the relative market entropy approach. First market entropy, measures the amount of information created every period, and the second one quantifies the speed of information digestion in the system. Building on the framework developed by Diebold and Yilmaz [2], they construct supplementary measures of market connectedness using information theory and population dynamics.

The first purpose of the present paper is to use these two methodologies developed by Diebold and Yilmaz [2] and Schmidbauer, Rösch and Uluceviz [8] with weekly return and weekly return volatility series and to interpret the different and similar outcomes of market connectedness in the case of using the expectation and the standard deviation of assets. The main goal is to analyze the market with respect to different structures, in our case, using the expectation of the prices and plus using the variations of them as a data. Using the spillover and entropy approach, we want to compare the results of market connectedness while return series is used and on the other hand, return volatility series is used instead of return. Analysing not only the return series but also the variations of them will give us an information about how do they complement each other. So, the main goal is to quantify dynamically the amount of connectedness of markets with a focus on the spillover and the flow of information and apply the methodologies using weekly return and weekly return volatility data. Therefore, this will give a chance to analyze different interpretations of market connectedness that comes from either using the expectation of the prices or using the variations of them. As it will be explained in Section 3.1.1, to compute the volatility spillovers we need to obtain the weekly volatility series. We developed a methodology using the Brownian motions idea. We simulate brownian motions to have an intuition for the estimation of volatility series. There are still limitations of our study, so the next attempt will be to solve the problems of this method and use this new method to obtain the volatility series and compute the volatility-to-volatility spillovers. Rather then the German and Klass approach we have also calculated the volatility spillover using daily volatility estimated by the GARCH
methodology. One of the purpose of this study is to compare the methodologies of volatility estimation and find the suitable one for our study. In the Empirical part, we compared the German and Klass and GARCH approach and analyse them in terms of volatility spillovers.

Second purpose of the present study, will be the investigation about the connectedness of Turkish stock market using an extended methodology of spillover index. We try to answer the question that which countries or organisations is Turkish stock market affected from. To do this, for now, we only analyse the return-to-volatility spillovers using the weekly return series of the stock market. We analyse the connectedness from four groups of countries to the Turkish stock market. For this purpose, we grouped the stock markets and not using them as a separate stock markets as before, we improved a new methodology, based on the principal component analysis, to omit the exaggerated influence of the large groups, since the groups are unequally large. We leave the theoretical explanation of this method as a future study of the thesis.

Interpreting the return spillovers will give us a chance to see the connectedness from the other groups of stock markets to Turkish stock market and the differences of this connectedness in the past decade. In the present paper, we will explain the results of the return spillovers and comment on them. We will also examine the from Turkey to Turkey connectedness to see the effect of itself. From the Section 5.2 we can conclude that the relation from Turkey to Turkey is increased in the last five years. In my future study, I will try to examine the reasons of this increase and find a specific economic or political events which causes this increase in the internal connectedness. The interpretation of the increase in the connectedness of Turkey and the Islamic countries will be also explained in the future studies. Then, we will also improve the methodology that we use for the case of Turkey and apply it using also the volatilities and not only the returns.

Third, as we explain the entropy methodology in Section 2.2 we will also use the KLIC and KS entropy idea in the case of Turkey and try to compare the method that we developed. Since, by using the entropy idea we have a feeling about the information that is produced by the system in every week and also the digestion period of this information from the system, we can utilise the entropy idea to interpret the reasons of the changes in the connectedness of the stock markets in the last 15 years.

The data that we used in this study divided into three parts to measure the connectedness with respect to two different goals. The first method used in the present paper is illustrated using data from the stock markets of five countries: Dow Jones Industrial Average (USA, New York Stock Exchange), FTSE (UK, London Stock Exchange), DAX (Germany, Frankfurt Stock Exchange), CAC40 (France, Euronext Paris), Nikkei 225 (Japan, Tokyo Stock Exchange). The weekly return and weekly return volatility data are used to apply spillover and entropy methodologies. R-project is used to apply the methodologies for both return and return volatility data.

The second one, uses daily closing quotations of the Russian stock index and the five stock indices representing stock markets in the "Systemic Five" countries. We will compute return spillovers using the same methodology with the first data set, but for
the volatility spillovers we will use the GARCH methodology instead of the German and Klass. The third one, the case of Turkey, uses 36 different stock markets of different countries where we grouped them into four parts with respect to their region or the organisations that they involve. The groups are: European union countries (EU), Organisation of the Islamic Conference countries (OIC), Brazil, Russia, India and China as in the group of BRIC, USA and Turkey itself.

This paper is organized as follows. Section 2, describes the methodologies of return spillovers and entropy. It is a review of the spillover methodology developed by Diebold and Yilmaz [2] and entropy methodology developed by Schmidbauer, Rösch and Ulucceviz [8]. Section 3 is the methodology of volatility spillovers which we develop and KLIC and KS entropy using volatility series. The method of obtaining volatility is explained using the idea developed by German and Klass [14]. The intuition of estimating the volatility series is explained in detail in this section. We have also explained the GARCH methodology of estimating the volatility series. Section 4 describes some properties of the three groups of data and explains how to obtain the weekly/daily return and weekly/daily volatility series for the first group. Empirical results of the two methodologies, explained in Section 2 and 3, are presented in Section 5.1 for both spillovers and entropy and with two estimation methodology of volatility spillovers and the comparison of them. The results of market connectedness for the case of Turkey using return spillovers is explained in Section 5.2. Section 6 concludes and discusses suggestions for further research.

2 Methodology I: Measuring return spillovers and shock repercussions

2.1 Return spillovers using forecast error variance decomposition (fevd)

Given a multivariate $N$ return series, the forecast error variance decomposition obtained from fitting vector autoregressive (VAR) models to windows of return data. This methodology can be briefly summarized as follows:

1. Fit a standard VAR (vector autoregressive) model to the series.
2. Establish an $n$ period ahead forecast.
3. Decompose the error variance of the forecast for each component with respect to shocks from the same or other components at time $t$.
4. Following Diebold and Yilmaz [2], for each market, arrange the fevd.
The decomposition of forecast error variance is given in terms of the structural VAR. A structural VAR model an order of 1 is in the form of

\[ B x_t = \Gamma_0 + \Gamma_1 x_{t-1} + \epsilon_t \]  

(1)

where \( \epsilon_t \) is white noise process. This equation can also be written as an MA representation:

\[ x_t = \mu + \sum_{i=0}^{\infty} (B^{-1} \Gamma_1)^i B^{-1} \epsilon_{t-i} = \mu + \sum_{i=0}^{\infty} \Phi(i) \epsilon_{t-i} \]  

(2)

where \( \Phi \) quantifies responses to shocks of size one standard deviation.

Then in the second step, the variance of the \( n \)-period-ahead forecast of \( x_l \) can be shown as

\[ \text{var}(x_{l,t+n} - \hat{x}_{l,t+n}) = \sum_{k=1}^{N} \sum_{i=0}^{n-1} (\Phi^k_l)^2(i) \]  

(3)

where \( (\Phi^k_l)^2(i) \) designates an impulse response function from series \( k \) to series \( l \). (the response of \( x_{lt} \) to a shock in \( \epsilon_{k,t-i} \), happening \( i \) time units earlier)

Next, in the decomposition of the error variance of the forecast, to omit the undesirable dependence on the ordering of markets, a generalized fevd is used, proposed by Pesaran and Shin [9]. They use Cholesky decomposition to identify the impulse response function of a component in the sense that they give the highest priority to that component. In other words: To identify the impulse response function of \( x_k \), use a Cholesky decomposition which allows \( x_k \) to have a contemporaneous impact on all other components \( x_1, \ldots, x_N \).

The fevd is then expressed in terms of the ratios

\[ \frac{\sum_{i=0}^{n-1} (\Phi^k_l)^2(i)}{\sum_{k=1}^{N} \sum_{i=0}^{n-1} (\Phi^k_l)^2(i)}, \quad l = 1, \ldots, N. \]  

(4)

The fevd gives the share of forecast variability in \( x_l \) due to shocks in \( x_k \), or, in other words, the return spillovers to volatility since return series is used in the model. After obtaining the fevd for each market, all spillovers can be arranged in the so-called spillover matrix as follows where \( N=4 \):

<table>
<thead>
<tr>
<th>from return ( x_k )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>□</td>
<td></td>
<td></td>
<td>□</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>■</td>
<td>□</td>
<td></td>
<td>■</td>
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<tr>
<td>( x_3 )</td>
<td>■</td>
<td>■</td>
<td>□</td>
<td>■</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>■</td>
<td>■</td>
<td>■</td>
<td>□</td>
</tr>
</tbody>
</table>

(5)

Each row thus sums up to 1 (or 100\%) and provides a breakdown of the forecast error variance of the corresponding stock index return with respect to shock origins in
terms of percentages. Each entry in the spillover table is called a directional spillover. Schematically, Diebold and Yilmaz [2] introduced the spillover index,

\[ \sum \square \sum \square + \sum \square \cdot \tag{6} \]

The network structure of the spillover matrix with respect to the propagation of shocks is a broad perspective, using the concepts from population, Markov Chain theory and information theory. The methodology developed by Diebold and Yilmaz [2] is used in the paper of Schmidbauer, Rösch and Uluceviz [8] where their goal is to extent the spillover idea and to find supplementary measures of market connectedness. More information about the spillover methodology can be found in Diebold and Yilmaz [2] and Schmidbauer, Rösch and Uluceviz [8].

2.2 Shock repercussions and Entropy

The starting point for shock repercussions is the spillover matrix on a given period. As it is described in the previous section row entries characterize the markets’ exposition to shocks while the propagation of the shock needs to be read column-wise. Due to the network structure of the spillover matrix, the population and Markov chain theory is used by Schmidbauer, Rösch and Uluceviz [8] to answer the following question: How are future volatilities across markets affected by a hypothetical shock hitting \( x_k \) on day \( t \)? How can we measure the strength of market repercussions of a shock?

Let \( M_t \) denote the spillover matrix for day \( t \). The propagation of the shock across the markets within day \( t \) can take place in a short time interval of unspecified length. The shock propagation (repercussion of the shock) can be modeled as

\[ n_{s+1} = M_t \cdot n_s, \quad s = 0, 1, 2, \ldots, \tag{7} \]

where a hypothetical shock “news”) of unit size to market \( i \) on day \( t \) can be denoted as \( n_0 = (0, \ldots, 0, 1, 0, \ldots, 0)' \). The index \( s \) denotes a hypothetical step in information flow, \( n_s \) characterizing what remains of the initial shock \( n_0 \) across markets after \( s \) steps. To investigate the steady-state properties they discussed the eigenvalue structure of the matrix \( M_t \). The left eigenvector \( v_t \), satisfying

\[ v_t' = v_t' \cdot M_t \tag{8} \]

called the “propagation values” of markets. The propagation value can be interpreted as the relative value of a shock to market \( k \) as seed for future variability in the markets. In other words it quantifies the strength of repercussions in the system of markets when a hypothetical shock originates from one of the markets.

The next discussion is about the location of the shock which can be explained using the transition matrices. The propagation values that is explained above can also be interpreted as stationary distribution of a Markov chain defined on the basis of a spillover
matrix. As given, a spillover matrix is not a suitable transformation matrix, because its rows sum up to 1 but its columns don’t. So, it can be changed by applying the transformation

\[ P_t = V_t^{-1} \cdot M'_t \cdot V_t, \]  

(9)

where the diagonal matrix \( V_t \) contains the left eigenvector \( v_t \) (corresponding to eigenvalue 1) of \( M_t \), and after re-scaling:

\[ \pi'_s = \frac{n'_s \cdot V_t}{n'_0 \cdot v_t}, \]

then the Markov chain equation emerges:

\[ \pi'_{s+1} = \pi'_s \cdot P_F, \quad s = 0, 1, 2, \ldots, \]  

(10)

The details about the transformation can be found in Tuljapurkar [13].

The equation can be interpreted as follows: On day \( t \), the initial location of a shock in the system is given by \( \pi_0 \) (a unit vector). The shock moves through the system according to the Equation (10). The stationary distribution of shock location is given by the vector of propagation values, which represents the “information equation” or “news balance” among markets on that day. Detailed information about the transformation and relation of them with the market connectedness, can be found in Schmidbauer, Rösch and Uluceviz [8].

The next questions can be as follows: How much information is produced by the system of markets from day to day? In other words how much information is gained from today’s to tomorrow’s (or next week’s in the case of using weekly data) news balance among markets? The question can be answered by applying the concept of Kullback-Leibler divergence (Kullback-Leibler information criterion, KLIC), which measures the entropy of day \( t \) with respect to day \( t-1 \), of the propagation values belonging to day \( t \) and day \( t + 1 \). So, the KLIC measures the relative variability of one probability distribution \( \pi_a \) with respect to the variability of a second distribution \( \pi_b \):

\[ \text{KLIC} = \sum_i \pi_a(i) \cdot \log_2 \left( \frac{\pi_a(i)}{\pi_b(i)} \right); \]  

(11)

In the concept of market connectedness, KLIC measures the initial information content of a shock (news) with respect to the news balance between markets in the long run. This idea is developed by Schmidbauer, Rösch and Uluceviz [8]. In cases where \( \pi_b \) characterizes the system of markets, KLIC is called the “relative market entropy”.

As it is explained at the beginning of this section, a hypothetical shock to a market will change the equilibrium, but then the market will “digest” the shock and reach the equilibrium again. How fast can the market converge back to the equilibrium after being hit by a shock? An appropriate measure for the speed of convergence is the Kolmogorov-Sinai (KS) entropy. Demetrius [1] introduced this entropy measure to population theory.
as “population entropy”; Tuljapurkar [13] relates it to the rate of convergence of a population. The rate of convergence to equilibrium defined as

\[ KS = -\sum_{i,j} \pi(i) \cdot \log_2 \left( p_{ij}^{ij} \right). \]

(12)

where \( p_{ij} \) denotes the entries in the transition matrix of the Markov chain as in the Equation (10) and \( \pi(i) \) are the stationary probabilities. Schmidbauer, Rösch and Uluceviz [8] examine the concept developed by Tuljapurkar [13] and adjust the definition in terms of market connectedness. More information about the methodology can be found in Schmidbauer, Rösch and Uluceviz [8].

3 Methodology II: Measuring volatility spillovers and shock repercussions

3.1 Volatility spillovers

Spillovers are important to understand the financial market interdependence. The spillover intensity is time-varying and this time-variation is fairly different for returns vs. volatilities.

The same fevd methodology is used, as it is in the Section 2, to obtain the volatility spillovers. The \( x_t \) in the Equation (2) that we meant return is now means volatility. So, we forecast the volatilities instead of returns. As in the return spillovers, where we need weekly return series, in this case we need weekly return volatility series to apply the VAR methodology for producing volatility spillovers. The methodology to obtain the return volatility series using the German and Klass’ formula and the intuition behind it is explained in Section 3.1.1 and the method of applying the formula to our data will be explained in Section 4.

3.1.1 Obtaining Daily Return Volatilities

There are different estimation methods to obtain the stock return volatility. We followed the estimation method of German and Klass [14] and try to understand the intuition behind it. We try to answer the question that why we can use the estimation method of German and Klass [14] instead of a GARCH methodology or can we modify the formula in a different way to estimate the volatility series. As, we can see from the Equation 15, the major difference of the two methodologies are, the German and Klass methodology uses only today’s information while GARCH uses also the previous information of the stock prices. The methodology of German and Klass [14] uses the historical opening, closing, high and low prices to estimate the volatility series. The model assumes that security prices are governed by a diffusion process of the form,

\[ P(t) = \phi(B(t)) \]

(13)
where $P$ is the price, $t$ is time and $\phi$ is a monotonic time independent transformation where we can obtain the maximum and minimum values of $B$ and $P$. $B(t)$ is a diffusion process with the differential representation

$$dB = \sigma dz$$

(14)

where $dz$ is a standard Gauss-Wiener process and $\sigma$ is an unknown constant to be estimated.

The methodology of German and Klass covers the usual hypothesis of the geometric Brownian motion of stock prices to estimate the return volatilities of the series. Although, the starting point of the estimation method is the Brownian motion, they mention some limitations of this methodology. They did modifications on the estimation method and use three different estimation methodology. The first one, uses only the opening and closing prices and the other two uses high and low prices in addition to the opening and closing prices. Using also high and low prices in the estimation model gives us more information about the data, since high and low prices during the trading interval gives us a continuous information about the prices changes while opening and closing prices are only ‘snapshots’ of the process. Finally, they found the formula below, which have the best efficiency factor among the three methodologies.

$$\hat{\sigma}_t^2 = 0.511(H_t - L_t)^2 - 0.019[(C_t - O_t)(H_t + L_t - 2O_t) - 2(H_t - O_t)(L_t - O_t)]$$

$$- 0.383(C_t - O_t)^2,$$

(15)

where $O_t$ ($H_t$, $C_t$, $L_t$) is the natural logarithm of the opening price in day $i$ (daily high, Friday closing, daily low) in day $t$. The methodology of how we implement this formula to obtain the weekly prices, not the daily ones, is explained in Section 4. For simplicity, we call the stock return volatility series as “volatility series” throughout the paper.

Explaining the coefficients of the Formula 15 is beyond the scope of this project for now. Instead of asking; “where does the coefficients come from?”, we want to show the intuition behind the formula. We develop some intuition to understand the methodology. We use the same approach like in the German and Klass methodology and simulate a Brownian bridge to understand the intuition behind this estimation method. Brownian Bridge is formulated as follows;

$$X_t = B_t^{\mu,\sigma} - tB_1^{\mu,\sigma}$$

(16)

where $B_t$ is a Brownian motion with $\mu$ and dispersion $\sigma$, and $0 \leq t \leq 1$.

Obviously, $X_0$ and $X_1$ are 0. So, we know the starting and ending points. In contrast to the Brownian motion we know the ending point of the process which is the closing price in our case. The idea behind simulating Brownian Bridges is that, if we can find a relation between the $sigma$ and the maximum and minimum values of the trading day then we can estimate the sigma, since we have the information about the high and low stock prices during the trading day. So, we try to answer the question; Can we
find a relation among the maximum and minimum values and the $\sigma$ value? In other words, is it possible to estimate the $\sigma$, using the maximum and minimum value of the stock price. First, we simulate Brownian Bridge where $B_t$s are normally randomized numbers. We simulate Brownian Bridges using different normally randomized $B_t$s. Then, we simulate the Brownian Bridge simulations for different $\sigma$ values. We have Brownian Bridge simulations with different normalized random Brownian motions. So, now we can obtain the maximum and minimum values of the Brownian Bridges for every simulation. Then, we plot the difference of ‘maximum-minimum’ values for different $\sigma$s to see that whether there is a relation among them. If we can find a relation, it means, we can estimate a $\sigma$ value by using the difference of ‘maximum-minimum’ values of a Brownian bridge. The graph of ‘maximum-minimum’ for different values of $\sigma$ is expressed in Figure 1.

As we can see from the Figure 1, there is a linear relationship between the ‘maximum-minimum’ value and the $\sigma$. So, if we know the maximum and minimum value of the Brownian Bridge (which in our case it will be the high and low prices of stock in day $i$) then using this information we can estimate the $\sigma$ value. Since, only the slope differ in the relation of ‘maximum-minimum’ values and $\sigma$, then we can specify an “average” slope to decide the relation and we can use this average slope to estimate the $\sigma$ value. The histogram of the slope values is given in Figure 2.
As we can see from the histogram the average of the slopes lies in the interval of (6,7). Obviously, this is not the value we can see in the Formula 15, but, at least it gives us an intuition of the methodology. We want to develop our study in the later versions of the thesis since now the intuition is missing at some parts. We have not answered some questions, for example, what if we have a drift together with the diffusion. In our study we always have a zero mean with different sigma values. So, the results will affect if we have a drift term with the volatility. Second, we want to find a way to estimate the sigma when the day is not finished due to an holiday or another reason. Now, to say something about the sigma our assumptions are made such that the day is complete and will finish at time 1. So, we will try to answer the questions above and improve our study to understand the methodology in detail.

3.1.2 Garch Methodology

We will compare the different methodologies of estimating volatility series and try to find the better estimation methodology for our study. For the first step, we will compare the German and Klass [14] methodology explained in Section 3.1.1 and the Garch methodology. As we mentioned in the previous Section, the Garch methodology uses today’s information as well as the previous day’s information of the stock prices. Since the methodology also affected by the previous days methodology, as we expected we found that the volatilities are generally higher than what we have found in the German and
Klass methodology. Dow Jones (USA) and Gdaxi (Germany) volatilities computed with German and Klass, and Garch methodology are in Figure 3. To examine which estimation methodology is better for the sake of our project we will analysis the methodology with the data in the Empirical part 5 and try to compare the results with respect to the volatility findings.

3.2 Shock repercussions and entropy for volatility

The market entropy is a tool to measure the amount of information, created from week to week. Comparing the return KLIC and volatility KLIC is very important to analyze especially the intervals where the behavior of the two are different. We will see in the Empirical part (Section 5) in detailed that there are different spikes in different times for return and volatility market entropies.

For the KS entropy, which is the speed of convergence back to the equilibrium after a shock, it is shown that return and volatility entropies have different characteristics for some intervals. Especially in the case of some specific events, the speed of information digestion varies in the case of return and volatility KS entropies. On the other hand, the KS entropy has another important characteristic. As it is discussed in Section 5 the overall pattern of KS entropy and spillovers are similar for both return and volatilities.

The same Markov chain approach is used, as in the Section 2.2, to produce the KLIC and KS entropy. The same methodology is used, which is developed by Schmidbauer, Rösch and Uluceviz [8], using the weekly volatility series obtained from Equation (15) instead of weekly return series.

4 Data

4.1 Data for Return vs. Volatility Spillovers - German and Klass approach

The two methodologies, spillovers and entropy, are used to see how markets are connected. For both return and volatility spillovers and also for return and volatility entropies, 15-year weekly stock market data of five countries are used. They are obtained from daily local-currency stock market indexes from 1997-01-06 to 2013-07-08 taken from Yahoo Finance\(^1\) and Datastream. The stock market indexes of five countries that are used, namely: Dow Jones Industrial Average (USA, New York Stock Exchange; in the following called dji), FTSE (UK, London Stock Exchange; ftse), Nikkei 225 (Japan, Tokyo Stock Exchange; n225), DAX (Germany, Frankfurt Stock Exchange; gdaxi), CAC40 (France, Euronext Paris; fchi). The level series are shown in Figure 4.

To obtain the weekly returns, we calculate the change in log price of close data, Friday-to-Friday. If the price data for Friday are not available due to a holiday, we use

\(^{1}\)http://finance.yahoo.com/
Figure 3: Comparison of Garch and German and Klass volatilities
Thursday. In the case where Thursday is still not available we use Wednesday. We proceed using this method until we reach an available data in the week. Figure 5 gives an impression of the dynamics of weekly return series as a percentage.

The formula, developed by Diebold and Yilmaz [2], that is explained in the Section 3 is used to estimate the weekly volatilities. Following Garman and Klass (1980) [14] and Alizadeh (2002) [15], the underlying daily high/low/open/close data is used to obtain weekly high, low, opening and closing prices to use the values in formula (15). For the weekly closing price the same method is used as in return: If available, Friday is used as weekly closing price, otherwise the previous available day is used. If available Monday is used as a weekly opening price, otherwise the next available day of the week is used. The highest value from Monday to Friday is used as weekly high price and the lowest value from Monday to Friday is used as weekly low price. Figure 6 gives an impression of the development of weekly volatilities.

4.2 Data for Return vs. Volatility Spillovers - using GARCH approach

The second empirical part consists of daily closing quotations of the Russian stock index(rts) and the five stock indices representing stock markets in the "Systemic Five" countries which we have used most of them in the Session 4.1: Dow Jones Industrial Av-
Figure 5: The series of weekly returns
Figure 6: The series of weekly volatilities
Figure 7: The level series of Systemic Five and Russia

The level series of Systemic Five and Russia

4.3 Data for the Case of Turkey

To investigate and measure the degree of international connectedness of Turkish stock market we use the return spillover methodology using stock markets of 36 countries and distinguish them to four different groups: European union countries (EU), Organization of the Islamic Conference countries (OIC), Brazil, Russia, India and China as in the group of BRIC, USA and Turkey itself. As it will be explained in detail in Section 5.2, we try to measure the connectedness of Turkey and compare the return spillovers to Turkey from this four groups in the last 10 years.
5  Empirical findings

5.1  Part I: Return Spillovers vs. Volatility Spillovers and Entropy

With the stock market data explained in Section 4.1 Section 4.2, market connectedness can now be obtained by applying methodologies I and II (Section 2 and Section 3) using weekly returns and weekly volatilities with the German and Klass methodology and daily return and daily volatility with the GARCH methodology.

5.1.1  Directional and overall spillovers -

Example 1: German and Klass methodology

In this section, we provide an analysis of five countries’ stock market weekly sequences of return and volatility spillovers (which means, weekly time series of spillover index values). First, we obtain the spillover matrices which resulted from fitting a sequence of VAR models along the steps outlined in Section 2 for return spillovers and Section 3 for volatility spillovers. Moving windows of 100 weeks and 5-week-ahead forecast are used in the construction of spillover matrices for both return and volatility. The spillover matrix which is shown in matrix (5) expresses the fraction of the forecast error variance of one country due to shocks to another country. The $ij$th entry of the spillover matrix means estimated contribution to the forecast error variance of country $i$ coming from shocks to country $j$.

Each entry in the spillover matrix is called a directional spillover. As an example of the directional volatility spillover, the US case is shown in Figure 8. As we can see the early 2000s the spillover that comes from US to the other markets is less than the spillovers going to US from others. But starting from 2004 until nowadays, on general, the spillovers that comes from US to the other markets are higher than the spillovers that come to US from others. It can be said that US is a net sender since mid-2004 in the sense that during the whole period spillovers coming from US are higher than those going to US.

Using the moving window of 100 weeks for fitting a sequence of VARs results in a spillover table for every week. Then using Equation (6) we assess the extent of spillover variation over time via the corresponding time series of spillover indexes. In other words the spillover table is an ‘input-output’ decomposition of the spillover index.

Overall return and volatility spillovers are displayed in Figure 9 and Figure 10, respectively, together with a smoothed version. Figure 9 is the return spillover from 1997 to 2013. The lowest spillover index is in October 2000 and it started to increase steadily to about 75% in October 2008 which is the peak date in the whole interval. After October 2008 it was in a decreasing trend and it currently stands at about 67%.

The volatility spillover graph can be seen in Figure 10. The lowest value in this case is in March 2006 but also in August 2000 the spillover is around 60% which is as low as
the minimum value of the spillover series. It started to increase after late 2006 until mid 2010 but unlike the return spillovers it decreased sharply. It currently stands at about 63%.

The comparison between return and volatility spillovers can be more precisely seen in Figure 11. Generally, volatility spillovers are higher than return spillovers until the end of 2006 and they only overlap in the period around mid 2006 which is also the time that volatility is at its lowest value. There are two time intervals that are attracting attention immediately by looking at the figure. From late 2008 to September 2010, both spillovers are in their peak values and move together. Similarly, starting from the beginning of 2011 both but especially volatility spillovers decreased sharply until nowadays. Specific events in these two time intervals have parallel impacts on return and volatility spillovers. On the other hand, there are other time intervals where one affected more than the other. Since, if the effect of a shock stays short it can be seen more in the volatility spillovers rather than in the return and in such a case it is hard to realize the effect of the shock in the return spillovers. In Figure 12 return spillovers are indicated in the x-axis and volatility spillovers in the y-axis.

In Figure 12, the comparison of return and volatility spillovers are shown. As it can be seen directly from the graph the values are positively correlated and the volatility spillovers are around 10% higher than return spillovers. There are also outlier values where specific events can explain the reason of this values. For example, the date of
terrorist attacks in U.S., September 11, 2001 is an outlier in the graph. The terrorist attacks launched by the Islamic terrorist group al-Qaeda, in the United States in New York City and the Washington, D.C. metropolitan area on Tuesday, September 11, 2001. The subsequent week September 17, 2001 the jump in volatility spillovers can be seen in the figure as an outlier. On 2001-09-17, the volatility spillover was 72.4% whereas the return spillover was 53.9%. Since, in the volatility spillover we look intra week high and low data, the spillover is reflected immediately by the change in the stock market prices due to a shock. The example of September 11 attacks, supports the idea above, the volatility spillover value jumped immediately and stayed high for three more weeks then again entered to the correlated association with return spillover. Unlike in the volatility spillover, there was no jump in the return spillovers. On 2001-09-10, the return spillover was 53.5% and after the attack it was still around 53.9%. Since, for the return spillover the weekly Friday-to-Friday return data is used, the shock might be absorbed by the return spillovers during the week.

September 15, 2008 is the day of Lehman Brothers bankruptcy, which is one of the largest bankruptcies in U.S. history. At the very day of this event, the volatility spillover was 74.6% and the return spillover was 69%. As it can be seen in Figure 12, September 15, 2008 is an outlier value and the effect of the event continues until the end of October. We cannot eliminate the other effects that might cause a jump in volatility spillover during the month, but at least Lehman Brothers bankruptcy induce a jump in volatility spillovers even if it might not be the only reason. On 2008-10-06, volatility spillover jumped and the following three weeks stayed high. On the other hand, until the end of September, return spillovers did not increase as much as volatility even though return spillovers are higher than their average value. So, again in the volatility spillover we observe a higher jump then the return spillover after the Lehman Brothers event.

Therefore, for these two specific financial events from the history, volatility spillovers are more sensitive to these shocks. The insight is that, many well-known events produced large volatility spillovers, whereas, with a few exceptions, none produced return spillovers. In the identified “crisis” events, volatility spillovers display immediate jump unlike the return spillovers.

5.1.2 Directional and overall spillovers - Example 2: GARCH methodology

The return spillover methodology is the same with the Section 5.1.1, the only difference is we have daily return series instead of weekly ones. For the volatility spillovers we have daily volatility series obtained by GARCH methodology.

The daily series of direct return spillovers from arts to others and from others to arts are displayed in Figure 13. We can say from the figure that spillovers to rts have always been higher than spillovers from rts. Return spillovers to rts rose sharply on 2014-12-17 the day after the Bank of Russia announced an increase of its key interest rate.

\footnote{http://www.businessinsider.com/largest-bankruptcies-in-american-history-2011-11?op=1}
Figure 9: Return Spillover

rate, the Russian weekly repo rate, from 10.5 to 17 percent as an emergency move to halt the collapse of the rule’s value. The big gap in from/to return spillovers lasted for about four months.

Propagation values quantify the relative importance of a market to act as a shock propagator. In other words, the propagation value measure the network repercussions of shocks. We will leave the theoretical part of the propagation value as a future study. Later versions of the thesis we will analyse the propagation value and more implementation of it. The propagation values of rts are displayed in Figure 14. The propagation values of rts were gradually increasing since 1997, peaking in early 2012, and have begun to decrease since then especially after 2014-12-17. When we compare the return spillovers and the propagation values especially after December 2014, we can conclude that direct spillovers tended to become more important, while network repercussions of shocks of arts decreased in importance.

For our second set of stock market examples we calculated the directional spillovers as we have done for example 1, for both return and volatility spillovers. So, with the daily volatility series that we obtained from the GARCH methodology we calculated the directional return and volatility spillovers for stock markets of six countries. Figure 15 displays the directional volatility spillover for the Russia case. The volatility spillover that comes from Russia to others is less than the spillovers going to Russia from others until the mid 2008. But stating from 2009 the spillovers to Russia from others increased
a bit and the effect from Russia to others became smaller. So, we can say that starting from the second half of the period, Russia is a net sender in the sense that during the period spillovers coming from Russia are higher than those going to Russia. But nowadays the spillovers from others to Russia is started to increase again and the net sender situation of Russia started to change.

The volatility spillover obtained by the GARCH methodology can be seen in Figure 16. The volatility spillover is always in an increasing trend but its peak value var at the beginning of 2007. It is started to decrease starting from 2012 and until today it is decreasing. It currently stands at about 35%. Generally, volatility and return spillovers are positively correlated with each other but volatility spillovers are higher than the return ones around 10%. The reasons for this difference will be examined in the later versions of this paper.

5.1.3 KLIC and KS entropy

- Relative market entropy (KLIC):
  The information that is produced by the system of markets from week to week is computed by Kullback-Leibler divergence as it is explained in the Section 2.2. KLIC, shown in Equation (11) where \( \pi_a \) denotes any initial distribution of a Markov chain and \( \pi_b \) its stationary (“news balance” in our case) distribution, provides a measure of how
distant the initial distribution is from equilibrium. The idea to use the KLIC as a relative market entropy in the field of market connectedness is developed by Schmidbauer, Rösch and Uluceviz [8]. From the point of market connectedness, KLIC measures the amount of information created week to week. Figure 17 shows the relative market entropy (KLIC).

The return KLIC, has an increasing trend until the end if 2004, but then the level of information created started to decrease. On the other hand, the return KLIC spikes have become shorter after 2008 except one big spike at the end of 2009.

In the volatility KLIC, always an increasing trend can be seen unlike the return. Except the big spikes, there are only small jumps in the volatility KLIC. The information that is created is high compared to an average level, especially for the following two cases: The volatility KLIC is at its highest value in 2011-03-21 and the second highest one is at the end of 2007.

Therefore, until the end of 2004 both return and volatility KLICs have an increasing trend but then unlike the volatility KLIC, return started to decrease. So, the magnitude of information gain has increased at the beginning for return but then decreased, whereas the volatility KLIC always increased. Also, in terms of the spikes, the information that is created week to week is higher in the case of volatility. The return KLIC spikes have became shorter after late 2008 and the volatility KLIC values had an increasing spike path, but the last spike was on March, 2011. The magnitude of the information created is huge for volatility but not for the return.
Figure 12: Return vs Volatility Spillovers
Figure 13: Return Spillovers from and to Russia

Figure 14: Propagation Value of rts
Figure 15: Directional Volatility Spillover from/to rts
Figure 16: Volatility Spillover with GARCH methodology
Second example of relative market entropy is in Figure 18 with a volatility KLIC. We can see big spikes especially mid 2010 and at the end of 2014. As we examined in the spillover part, 2014-12-17 the day of increasing the interest rates of Russian bank we also see the effect of the day in the KLIC. The magnitude of the information gain increase immediately after the day and a few days later it dissapears.

Figure 19 shows the relation of return and volatility relative market entropy values. There is a linear association between return and volatility KLIC values. The log values are used for the transformation of scale. As it is seen in the graph, even the volatility KLIC values are in the wider interval than the return KLIC, there is a linear association between two of them. So, we can say that there is a positive correlation between return and volatility entropy values. Is it possible to find an example where the correlation is not positive? We leave this question for future research and try to find an either real or
Figure 18: Volatility relative market entropy for the case of Example 2 (with Russia)
hypothetical example where the correlation is not positive.

- **KS entropy:**
  
  There is another type of entropy measure which is explained in Section 2.2 as a Kolmogorov-Sinai entropy measure. Demetrius [1], introduced this entropy measure in population theory and Schmidbauer, Rösch and Uluceviz [8] adjust it in the field of market connectedness. The idea that is developed by Schmidbauer, Rösch and Uluceviz [8] is the answer to this question: How the speed of convergence to a news balance between markets changed across time if the system is distorted by a shock (news)? In other words, KS entropy measure the speed of information digestion. The KS entropy for return and volatility is shown in Figure 20.

  The overall patterns of KS entropy and spillover index are similar, both for returns and volatilities. This relation can be seen by comparing the Figure 20 and Figure 11. The volatility KS entropy is higher than the return KS entropy until 2006, then they overlap. We have seen a similar behavior in the case of return and volatility spillover indexes. Both return and volatility KS entropies are in their peak values on October 2008 until September 2010 and then both of them decreased sharply. The similar peak dates can be seen in the case of return and volatility spillover indexes.

  Figure 21 shows the pattern of return and volatility KS entropies. The points show the return KS entropy in the x-axis and volatility KS entropy in the y-axis. There is a positive correlation in the return and volatility KS entropy, and volatility KS entropy value is generally higher than the return KS entropy.

  The specific events that we have discussed in Section ?? in terms of the spillover perspective can also be adopted to the KS entropy for returns and volatilities. There are similarities between Figure 21 and Figure 12. Some outlier values are in similar dates in KS entropy and spillover index.

### 5.2 Part II: Market Connectedness: The Case of Turkey

We show in Section 5.1 how to apply the methodology of return and volatility spillovers using the five stock market. In this section, our goal is to present an investigation about the connectedness of Turkish stock market and to measure it using a new methodology than in Section 2. For now, we only look at the return-to-volatility spillovers using the weekly return series of the stock market. We leave the volatility spillover in case of Turkey part as a future work of this thesis.

Here, instead of using the stock markets alone we have grouped the stock market of 36 countries in terms of an organizations as follows: Turkey, European union countries (EU), Organization of the Islamic Conference countries (OIC), Brazil, Russia, India and China as in the group of BRIC, and USA. So, we only leave Turkish stock market alone to measure the connectedness with the other groups.

The aim of doing this is we try to assess the degree of Turkish connectedness with other groups of markets, in particular with the OIC countries. Although we use the same return spillovers methodology like in Section 5.1, we decided to develop the methodology,
Figure 19: Return vs volatility relative market entropy
Figure 20: Return and volatility KS entropy

because there are problems while we use different stock market groups instead of taking every market separately. We again consider a directed network with equity markets as nodes and return-to-volatility spillovers, obtained via forecast error variance decompositions (fevd) as the weights of the edges. But, when other markets are grouped together (such as stock markets within the EU), it is not meaningful to simply add up percentages across markets in each group, because groups are unequally large, the number of countries is not the same in every group, and so by simply adding up percentages leads an exaggeration of large groups’ influence. Thus, we developed the methodology and first summarize the series within each group using principal component analysis and subsequently decide which series to include by comparison with white noise. We leave the theoretical detailed explanation of the developed methodology as a next study of the thesis. In Figure 22 we showed the return spillovers to Turkey from four different groups that we mentioned above and Turkey itself.

So, we can see that last ten years the return spillovers from Turkey to Turkey itself increased a lot in the last five years. It means the share of spillovers originating from outside to the local stock market is decreasing and Turkey is affected more from itself. We can also conclude that the spillover from OIC to Turkey is also increased which might be interpret as Turkey started to be more connected to the Islamic countries than before. As a future study, using the notions related to the network centrality and add information entropy to this study, we can also identify political and economic events
Figure 21: Return vs volatility KS entropy
having a big impact on the distribution and origin of stock market volatility in Turkey.

6 Conclusion and Further Research

The first goal of the present study was to compare the results of market connectedness using the spillover and information flow perspective when the return prices and volatilities of them are used. Using both expectation of stock prices and the variations of them, gave us an information about how these two complement each other. So, we analyzed the market with respect to different structures.

We applied spillover and entropy methodologies on expectations of prices and variations of them. Firstly, we compared the return spillovers and volatility spillovers. We reached the result that even the return and volatility spillovers behave similarly and there is a positive correlation among them, at some specific time intervals return spillover and volatility spillover act different. We tried to explain these results in terms of a “crisis” or an important event from the history.

The results obtained from KLIC and KS entropy showed us the information gain comes from either using the return or volatility series. In the KLIC results, the amount of information created week to week had different pattern in the case of return than the volatility. The spikes from the volatility is higher than the spikes coming from return. The magnitude of the information created has been increasing for volatility but in the
return there are time intervals where information created has been decreasing.

There is a positive correlation between the values of return market entropy and volatility market entropy. In the further research of this study we will examine the reason of this correlation and try to find an example where there is no correlation or the correlation is negative. Since, it is not obvious that we can find a real example for this, we will construct a hypothetical example where the correlation is negative among the return KLIC and volatility KLIC. Therefore, we can compare the information created in these two cases where there is positive and negative correlations.

In the case of KS entropy, the similar situation like in the spillover index have found. There is a relation in the return and volatility KS entropies, but at some specific time intervals the volatility KS entropy, which gives us an information about the shock digestion, is higher than the return KS. We tried to explain the reasons as we did in the spillover index. We will improve the entropy idea in the future study of the thesis and examine the results in terms of economics events. Also, we will try to find a link among the spillover methodology and the entropy.

Then, since we have used the weekly return and weekly volatility series, we needed to obtain the weekly return and volatilities using daily data. We followed the methodology of German and Klass [14] to construct the weekly volatility series and tried to understand the intuition behind the method. So, both spillover and information flow perspective depended on the weekly return and volatility series that we have obtained. We have also used the GARCH methodology to obtain the volatility series and we compared the results with German and Klass methodology. We have used a new stock market data group to examine the different effects and compare the two methodologies calculating the volatility spillovers. In our further study, we will develop the idea and also try to find the best methodology of volatility estimation for our study.

The second goal was to measure the connectedness of Turkey using a new methodology of return spillovers. We conclude that the spillovers from Turkey to Turkey increased in the last five years. We will interpret the reasons of this increase in the future studies and try to find an economic interpretation of them. We will also examine the change in the relation between Turkey and Islamic countries based on the OIC countries. After completing the return spillovers part we will apply the new methodology in the case of Turkey using the volatility spillover method. This will be an important contribution since due to the different interpretations of return and volatility spillovers we will understand the change in the connectedness in Turkey using the stock market and interpret the Turkish economy in the last decade. We will also add the entropy methodology in addition to the spillover one and try to examine the market and understand the effects of information and the digestion of them in the economy.

We would like to mention that the present paper have been submitted and accepted for presentation at the Ecomod conference, in Boston, United States, in July 2015.
References


